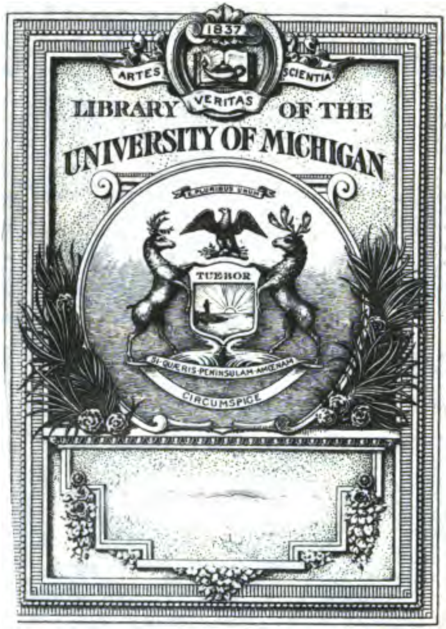
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ELEMENTS OF ASTRONOMY.

ELEMENTS OF ASTRONOMY.

John BRINKLEY'S

ASTRONOMY.

REVISED AND PARTLY RE-WRITTEN,
WITH ADDITIONAL CHAPTERS,

BY

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TOGETHER WITH AN APPENDIX OF QUESTIONS FOR EXAMINATION.

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PREFACE.

THE "Elements of Astronomy" of Bishop Brinkley has for more than sixty years been used as a text-book by the Students of the University of Dublin. When Dr. Brinkley held the office of Andrews' Professor of Astronomy, he delivered the substance of this work, in his public annual prelections to the Undergraduates, from the year 1799 to 1808. In the latter year, at the request of the Board of Trinity College, he printed these lectures in a volume, and afterwards added an appendix of problems. Since that time the work has formed an important portion of the course of study prescribed for the Undergraduate Lectures and Examinations, yet very little alteration had been made in the text to correspond with the recent progress of Astronomical Science. The author had intended to prepare some additions to the work, particularly on the subject of double stars and comets, but was prevented by his episcopal duties, and a distressing and tedious illness. It was long felt by those who were engaged in the teaching and examining of

the Students, that while the plan of the treatise was excellent, and admirably adapted for the intellectual training of the Undergraduates, it was in many respects seriously defective as a text-book.

There were many points in the reasoning which, although sufficiently clear to a thinker of Dr. Brinkley's ability, required much further elucidation before they could be made intelligible to the ordinary Student. Very important subjects connected with the science were omitted altogether, and in recent times it was universally felt that the treatise was not in keeping with the advanced state of Astronomical Science. The Provost and Senior Fellows of Trinity College, having had reason to believe that the work required considerable improvements, if it were to be continued as a text-book for all Students, and not wishing that a treatise so long and intimately connected with the studies of the University of Dublin should be superseded, commissioned me to undertake a thorough revision of it, and to endeavour to make it suitable to the requirements of the Students and to the present advanced state of the Science.

For this purpose I placed myself in communication with my friend Dr. Brünnow, the present eminent successor of Dr. Brinkley in the chair of Astronomy; and the valuable assistance which

he promptly afforded very much aided me in my attempt to produce a text-book which a long experience as College Tutor enabled me to see would meet the wants of the Students.

The whole of Dr. Brinkley's work has been carefully revised, corrected, and in many places altogether re-written. Some considerable portions of the book which were found to be unsuited to the Students have been omitted, while new chapters and portions of chapters have been added which are not to be found in the original work.

The arrangement of the chapters has been considerably altered; those which are required of all Students, at the ordinary Term and Degree Examinations, have been placed together. I have written a chapter on the masses of the heavenly bodies and on the Tides, and have also supplied an account of the principal comets, and a method of treating the equation of time different from that adopted by Brinkley, together with numerous other minor additions to every chapter.

Dr. Brünnow, besides giving general suggestions, has written new chapters on the physical constitutions of the sun and heavenly bodies, on the discoveries by means of the Spectroscope, on the proper motions of the fixed stars, on the recent methods of ascertaining the parallax of the fixed stars, and on the general advance of Stellar Astronomy. He has also remodelled the portions of the work which

treat of instruments, and made them suitable to the present improved condition of practical Astronomy.

I have also endeavoured to substitute new and simpler mathematical demonstrations in many cases where my experience led me to think that the old ones were too cumbrous, and have added a series of questions on the first thirteen chapters which I hope will be found useful to the Students preparing for examinations.

On the whole, it has been my anxious wish to remodel the work of Bishop Brinkley so as to make it correspond with the present extended knowledge of this science, and at the same time to furnish in it such a text-book as my experience as a teacher shows to be required.

I wish to return my best thanks to Professor Brünnow for his assistance, and also to my friend Mr. Williamson, for his trouble in reading the proof-sheets, and for his kind suggestions.

JOHN W. STUBBS.

TRINITY COLLEGE, NOVEMBER, 1871.

INTRODUCTION.

The Science of Astronomy has advanced to its present state, by means of a series of observations and discoveries made during a long course of ages. We can now select from these such as will best conduce to demonstrate the true system, and explain the various phenomena.

Astronomy, by making known to us the immensity of the creation, necessarily increases our reverence of the Divine Creator. This, alone, is a sufficient reason for making it a part of general education. It also, perhaps, furnishes a more satisfactory application of the abstract sciences than any other part of Natural Philosophy. Its practical utility is also considerable. It has always been useful in Geography and Navigation, and within the last century has afforded splendid assistance to the latter, by the lunar method of finding the longitude at sea.

When the student first applies himself to subjects of Natural Philosophy, it is of much importance that he should proceed by the same strict and accurate manner of investigation to which he had been accustomed while engaged in the rudiments of Mathematics.

The perfection of modern instruments, and of modern observations, admits of an arrangement which will afford, with respect to the most important facts in Astronomy, nearly the same degree of conviction to the mind as it receives from the elements of Euclid, and which requires little more preparatory knowledge. Such an arrangement has here been had in view.

The phenomena of the celestial bodies observed by a spectator fixed in one place are first noticed. The uniform apparent diurnal motion of the concave surface, carrying with it the sun, moon, planets, and fixed stars, leads to the definitions of the celestial equator, poles, meridian, declination, &c. The considerations of the apparent motions of the sun, moon, and planets, on the apparent concave surface, lead to the definitions of the ecliptic, of right ascension, longitude, &c. The various problems of the sphere have their origin from the apparent motion of the concave surface and the apparent motions of the sun, moon, and planets on this surface. This is almost all the astronomical knowledge that could be attained to by a spectator fixed to one spot, and not possessing observations made in distant places. He could form no accurate notions of the actual magnitudes, and actual distances of the sun, moon, and planets. All the certain astronomical knowledge that existed for many ages was limited to the doctrine of the sphere.

The next consideration is, in what manner we can ascertain the actual magnitudes of the celestial bodies and their actual distances from us. Telescopes enable us to examine more exactly their appearances, and serve

to exhibit many most interesting phenomena, but do not directly lead us further.

The first step of importance is a knowledge of the form and magnitude of the earth. The fixed stars appear in the same relative situations, at the same angular distances from each other, and from the visible celestial pole, in whatever part of the earth we are. The most exquisite instruments point out no alteration. The conclusion drawn from this is, that the fixed stars are at distances so great, that lines directed from all places on the surface of the earth towards the same fixed star, or towards the visible celestial pole, must be considered as parallel. Combining this with what has been observed in so many places, that the variation of altitude of the celestial pole is proportional to the space gone over in a direction north or south, and that for a change of altitude of one degree the space is about 69½ miles, it is easily proved that the earth is nearly a sphere of about 8000 miles in diameter.

This is an important step: we thus ascertain that a space of 8000 miles is as nothing compared with the distances of the fixed stars.

It also follows that the altitude of the celestial pole is equal to the latitude of the place. This conclusion enables us to solve the problems arising from the situation of the celestial circles in different places, and to explain the variety of seasons over the whole earth, independently of the knowledge of the true system.

Having ascertained the form and magnitude of the earth, the next step is to investigate the magnitudes of the sun and planets, or at least to show that some of

them greatly exceed the earth in magnitude, and also to show their vast distances compared with the diameter of the earth. It is important that this should be done previously to demonstrating the true system.

Certain observations, made with micrometers, at two places considerably distant from each other, but nearly under the same meridian, serve for this purpose. The student will readily apprehend this method; he will see, that by it we are enabled to ascertain the angle the disc of the earth would be seen under, could we remove ourselves to a planet to make the observation. This angle can be ascertained with as great precision as we can measure the apparent diameter of a planet seen from the earth. If, with respect to some of the planets, the angle which the earth's disc subtends be so small that it is within the limit of the errors of observation, yet we obtain a limit of the magnitude of the earth compared with the magnitude of the planet. Thus the earth seen from Jupiter subtends an angle of four seconds, when Jupiter seen from the earth subtends an angle of forty seconds. Now, if it be contended that neither of these angles can be ascertained to a second or two, it will make no difference as to the purpose for which this mode of ascertaining the relative magnitudes of the earth and Jupiter is introduced. It will sufficiently show that the magnitude of Jupiter greatly exceeds that of the earth, and also will show that the distance of Jupiter is many thousand times greater than the diameter of the earth.

The spots upon the sun, and appearances in several of the planets, show that they are spherical bodies, hav-

ing a motion of rotation on their axes. All this, being quite independent of any hypothesis as to the arrangement of these bodies, assists much in the arguments by which the rotation of the earth on its axis, and its annual motion round the sun in an orbit nearly circular, may be proved.

The different motions of the planets on the concave surface which appear so irregular are chiefly explained by their moving in orbits nearly circular about the sun.

By following an arrangement of this kind any student may, without difficulty, satisfy himself of the truth of the Copernican system. He will find this manner of treating the subject pursued in the first seven chapters of this work.

After the true system has been explained, the subsequent arrangement in a treatise on astronomy seems of little consequence.

In this work, after the motions of the primary planets are explained in a general manner, the motions of the moon and secondary planets, and several other circumstances connected therewith, are briefly noticed.

A short account of instruments and observations, by which the places and motions of the celestial bodies are exactly ascertained, is followed by a more exact statement of the planetary motions, and by an account of Kepler's discoveries; also, by a more particular account of the motions of the moon, of the satellites and of comets.

Several of the phenomena which arise from, or are pointed out by, the motions and bodies of the solar system, are next considered. Such are the eclipses of

the Sun and Moon, the transits of Venus and Mercury over the Sun's disc, the velocity and aberration of light, and the equation of time.

The application of astronomy to navigation and geography is also introduced, and the importance of the former has occasioned a rather long detail.

A chapter on the method of ascertaining the masses of the earth, sun, and planets, and the effect of the attractions of the sun and moon on the waters of the sea in the production of the Tides, has been added; also, detailed accounts of the recent methods of investigating the physical constitutions of the sun and other heavenly bodies, and of observing the annual parallax of the fixed stars.

The chapter on the discoveries in physical astronomy contains little more than an historical account. It had been at first intended that it should contain the elementary parts of physical astronomy, as far as respected Kepler's discoveries. Physical and plane astronomy are now so connected that it is difficult to treat of them separately.

Facts in the history of astronomy have been only occasionally introduced. The student, who had made himself so well acquainted with astronomy as to find its history interesting, will easily procure for himself, from a variety of authors, all the information he can desire.

Among the various advantages derived from the science of astronomy, there is one eminently deserving of notice. We see the most complex appearances and most intricate apparent motions admitting of the simplest explanations.

How intricate and various are the apparent motions which depend only on a primary motion of projection and the simple law of gravity! This may assist us in other departments of natural science, and may encourage us to expect that the most difficult phenomena may at last be found to arise from the most simple laws.

The aberration of light furnishes a remarkable illustration.

Light moves about 200,000 miles in a second; had it moved only 50 miles in a second, it is probable that astronomy would not now have existed as a science. The motions of the stars and planets would have appeared inextricable confusion. The face of the heavens would have been continually changing, and could not have divided into constellations. Stars which at one time would be seen close together, at another would appear many degrees asunder. All this would be occasioned by the simple change of the velocity of light, and, as is easily understood, would arise from a combination of the motion of light, and of the other motions in the system. If this motion be pursued in all its bearings, it cannot be doubted that a consequence of such an alteration in the velocity of light would have been, that this science, by which our knowledge of the creation is so much extended, would scarcely as yet have existed.

CONTENTS.

CORRIGENDA.

The reader will kindly make the following corrections:—

Page 47, line 17, *for* ECT, *read* ECS.

—— ,, line 18, *for* $(l + l)$, *read* $(l + l')$.

—— 70, *the termination of line* 18 *from bottom should read* this uniform, *and of line* 19, pole of the.

—— 112, line 19, *for* come, *read* curve.

—— 113, lines 20 and 21, *for* Bengenberg and Brundes, *read* Benzenberg and Brandes.

—— 114, lines 14 and 25, *for* Temple, *read* Tempel.

—— 114, line 25, *for* consists, *read* is part.

—— 203, line 7, *for* $\frac{\sqrt{1+e}}{1-e}$, *read* $\sqrt{\frac{1+e}{1-e}}$.

ELEMENTS OF ASTRONOMY.

CHAPTER I.

ON THE DOCTRINE OF THE SPHERE.

1. THE imaginary concave surface in which a spectator at first conceives all the heavenly bodies placed is an hemisphere, in the centre of the base of which he himself is situate. The base of this hemisphere is the plane by which his view of the heavens is bounded. It is called the *plane of the horizon.*

The numerous bodies observed on the concave surface differ in lustre, and apparently in magnitude. All of them appear to have a daily motion. Many of them emerge, as it were, from below the plane of the horizon, and after traversing the concave surface, disappear, to rise again at the same points of the horizon as before. Others in their paths never reach the horizon, but continually move round a fixed point in the heavens.

Far the greater number of the celestial bodies preserve the same situation with respect to each other; that is they preserve the same apparent distances from each other. These are called *fixed stars.*

The sun, besides his diurnal motion of rising and descending, seems also to have a motion on the concave surface, and in a certain space of time, called a year, to return to the same position with respect to the fixed stars.

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The moon appears also, besides its diurnal motion, to have a motion among the fixed stars, and in a space of time called a month, returns nearly to the same position with respect to the sun.

2. The spectator viewing those stars that do not set, will observe one of them nearly immoveable. This is called the Polar Star, from its vicinity to the point about which the stars that do not set appear to move. The point itself is called the *North pole.*—The face of the spectator being turned to this point, the stars rise on his right hand, or in the east, and set on his left hand, or in the west; and thus the apparent diurnal motion of the celestial bodies that rise and set, is from east to west.

The apparent motions of the sun and moon among the fixed stars, are in a contrary direction; that is, from west to east.

Besides the sun and moon, and fixed stars, a number of other celestial bodies may be noticed, which, beside their apparent diurnal motions, have apparent motions that at first seem not easily brought under any general laws. Sometimes they appear to move in the same direction among the fixed stars as the sun and moon; at other times in a contrary direction, and then are said to be retrograde. At times they appear nearly stationary. The larger of these bodies are named planets. They have been named Mercury, Venus, Mars, Ceres, Pallas, Juno, Vesta, Jupiter, Saturn, Uranus, and Neptune. Of these, five have been noticed from the remotest antiquity. Of the remaining two, which are only visible by the assistance of telescopes, Uranus was discovered by Sir W. Herschel in 1780, and Neptune in 1845 by Dr. Galbe, of Berlin, nearly in the place predicted by the independent researches (on the observed irregularities in the motion of Uranus) of Le Verrier, of Paris, and Professor Adams, of Cambridge. These seven constitute the great planets. Besides these there is a group of minor planets, called Asteroids, which (with the exception, perhaps, of one or two) cannot, even with a telescope, be distinguished from fixed stars, except by their motions. Of these the four brightest were dis-

covered in the beginning of the present century; Ceres, by Piazzi on the 1st January, 1801; Pallas, at Bremen, by Dr. Olbers; Juno, at Lilienthal, by M. Harding; and Vesta, by Dr. Olbers; the next, Astræa was discovered by Hencke in 1845; and since that time several new ones have been noticed; so that at present (1871) their number amounts to 113. The large planets, and some of the Asteroids, are always found to be near the annual path of the sun in the concave surface; but many of the latter are often at a great distance from this path. Two of the larger planets, Mercury and Venus, are never far from the sun.

3. The above are a few of the phenomena which offer themselves in contemplating the heavens. But the motions are in general only apparent, and take place from a combination of a variety of different motions. The difficulty of deducing the actual circumstances of the magnitudes, and of distinguishing the true from the apparent motions of these bodies, however easy it may appear when done, is such that we ought not to be surprised that the ancients made so little progress toward the knowledge of the true system and true dimensions of the universe; nor ought we to think lightly of their efforts, and to treat them with contempt for their errors. The moderns, by the joint assistance of mechanics, optics, and mathematics, have advanced the science of astronomy to a greater degree of perfection, perhaps, than any other branch of natural knowledge.

4. For more readily explaining and referring to the phenomena of the celestial bodies, certain circles are imagined to be described on the concave surface. Distances on the concave surface are measured by arcs of great circles, or circles of a sphere, the plane of which passes through the centre. The[a] present division of

[a] The old mode of expressing the measure of an arc was by stating its relation to the whole circumference; thus the diameter of the sun, measured on the concave surface, was said to be $\frac{1}{720}$ of a great circle.

the circle into 360 equal parts, called degrees, of each degree into 60 equal parts, called minutes, and of each minute into 60 equal parts, called seconds, was not used till long after astronomy had attained to a considerable degree of perfection.

It is much to be regretted, that, at the revival of learning in Europe, a decimal division of the circle was not adopted, which would have greatly facilitated astronomical computations. The French, after the Revolution, adopted this division for a short time, but not generally. They divided the circle into 400 parts, each quadrant containing 100, each of these parts into 100, &c. We find this division adopted in some books of that period; for instance, the "Mecanique Celeste" of Laplace.

The circles, and arcs of circles, forming parts of the instruments used in practical astronomy, are actually divided into degrees and parts of a degree, as far as the magnitude of the radius will permit, so that the divisions may not be too close together. The arcs of limbs of the larger astronomical quadrants and circles are divided into intervals of five minutes, and those of the largest circles into intervals of two minutes. But the measure of an angle can be obtained to $\frac{1}{10}$ of a second by the assistance of ingenious contrivances, that will be noticed hereafter. The most improved instruments are thus adapted to measure angles to fractional parts of seconds. In general, with the best instruments, the result of a single observation can now be depended on to a second, and in many cases to a fraction of one second.

5. Let us return to the consideration of the visible concave surface of the heavens. The intersection of the plane of the horizon, with the imaginary concave surface, is a great circle, which may be called the *celestial horizon*. A plumb line hanging freely and at rest, is perpendicular to the plane of the horizon, and a small fluid surface at rest is in the plane of the horizon. These two circumstances are of the utmost importance to the practical astronomer. The impossibility of having, except at sea, an uninterrupted view, and other causes, make

it difficult for him to use the horizon itself; but the plumb line and fluid surface fully compensate for these inconveniences.

The *altitude* of a celestial object is its distance from the horizon, measured on a great circle passing through the object, and at right angles to the horizon. Such a circle is called a secondary to the horizon; a great circle at right angles to another great circle, being called a *secondary* circle. And the *zenith distance* of a celestial object is its distance from the upper pole of the horizon, which is called the *zenith*. By the assistance of a plumb line and quadrant, the altitude or zenith distance may be readily found. Let ACQ (Fig. 1.) be an astronomical quadrant, the arc AQ of which is divided into degrees, &c., the radius AC is adjusted perpendicular to the horizon, by turning the quadrant about the point C, till a plumb line, suspended from C, passes over a point A. The radius CQ is then horizontal. A moveable radius or index CT is placed in the direction CO of the object, by means of plain sights at the extremities of the radius C and T (now rarely used), or by means of a telescope affixed to the radius. The arc TQ will then show the altitude, for TCQ equals HCO the altitude; and the arc TA will show the zenith distance, for ACT equals OCZ the zenith distance. The method of observing altitudes will be more accurately described hereafter: it was thought necessary to advert to it here; and also to mention how an angular distance on the concave surface may be measured.

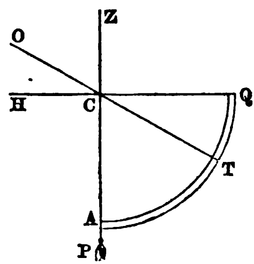

Fig. 1.

A circle divided into degrees, &c., furnished with a fixed radius, and a moveable radius, being placed in the plane passing through two objects and the eye, the circle may be turned till the fixed radius passes through one object, and then the moveable radius being made to

pass through the other, the arc intercepted between them will show the angular distance. This method is now rarely used. The angular distance of two objects when required, is seldom *directly* observed, on account of the inconvenience of adjusting the plane of the instrument, and the two radii, to two objects, both of which perhaps are moving, and with different motions. Therefore, in this way, great accuracy cannot be attained: but the conception of this method, although inaccurate, will be useful in what follows. When an angular distance on the concave surface is required, it is generally obtained by computation from other observations, *e.g.* from the declinations and right ascensions (to be explained hereafter). In one instance, indeed, *in the lunar method of finding the longitude*, it is necessary to observe, with great precision, the distance of the moon from the sun or a fixed star. This is done in a manner hereafter described, by an Hadley's sextant, an invaluable instrument for the purpose. By this instrument also the angular distance between any two objects may be measured.

6. To explain the phenomena of the apparent diurnal motions of the celestial bodies, we imagine an hemisphere below our horizon, and in it a point diametrically opposite to the north pole, which we call the south celestial pole; we also imagine that the concave surface turns uniformly on an axis, called the axis of the world, passing through the north and south poles, completing its revolution in the space of $23^h\ 56^m$ nearly, carrying with it the sun, moon, and stars, while the horizon remains at rest.

This hypothesis illustrates and represents the apparent diurnal motion of the several celestial objects in parallel circles, with an equable motion, each completing its circular path in the same time. That the motion of each star is equable, and that they describe parallel circles on the concave surface, we deduce from observation and the computations of spherical trigonometry. This will be readily understood from what follows.

The great circle, the plane of which is at right angles

to the axis of the world, is called the *Equator*. Since all great circles of a sphere bisect each other, this circle is bisected by the horizon, and therefore all celestial bodies situated in it are, during equal times, above and below the horizon; consequently when the sun is in this circle, day and night are of equal length, whence it is also called the *equinoctial.*

This representation of the diurnal motion, by the motion of a sphere about an axis inclined to a plane representing the horizon, on which sphere the celestial bodies are placed at their proper angular distances, must have been among the first steps in astronomy. Yet in the infancy of the science, doubtless, a considerable time elapsed before it was known that the diurnal paths of the stars were parallel circles, described with an equable motion. Without this, little progress indeed could have been made. It is likely that at first it was little more than an hypothesis, in some degree confirmed by the construction of a sphere, to represent by its motion the celestial diurnal notions; for its confirmation, by the application of spherical trigonometry, seems to require a greater knowledge than we can suppose then existed.

This diurnal motion, we now know, is only apparent, and arises from the rotation of the earth about an axis, by which the horizon of the spectator revolves, successively uncovering, as it were, the celestial bodies, while the circles of the sphere are at rest. But the phenomena are the same, whether the horizon is at rest and the imaginary sphere revolves, or the horizon revolves and the imaginary sphere is at rest. By conceiving the sphere to revolve and the horizon to be at rest, the phenomena are more easily represented. Four centuries since, this apparent diurnal motion was generally considered to be real; and had we not the knowledge derived from navigation, and the communication of observations made in distant countries, we might still contend for the truth of it. Now we only imagine it, for more readily explaining the phenomena of the sphere and the circles thereof.

7. *Circles of the sphere.*—Secondaries[a] to the equator are called *circles of declination,* because the arc of the secondary, intercepted between an object and the equator, is called its *declination north* or *south,* according as the object is on the north or south side of the equator.

The great circle passing through the pole and the zenith, is called the *meridian.* This circle is at right angles, or a secondary, both to the horizon and equator. It is easy to see that it divides the visible concave surface into two parts, eastern and western, in every respect similarly situate as to the pole and parallel circles. The eastern parts of the parallel diurnal circles being equal to the western, and the motions equable, the times of ascent from the horizon to the meridian, are equal to the times of descent from the meridian to the horizon.

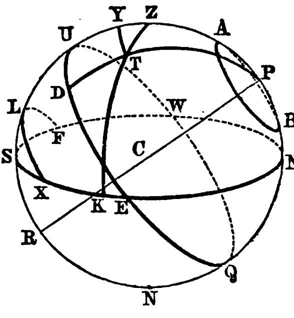

Fig. 2.

In (Fig. 2) the circle WSXKEN represents the horizon, the centre C of which is the place of the spectator. The part of the figure above this circle represents the visible concave surface; and the part below, the invisible. Z is the zenith; P the visible, and R the invisible pole. PZUSRNQ is the meridian, EQWU the equator. AB a small circle parallel to the equator, FLX the visible portion of another parallel to the equator. A star, situate in AB, is continually above the horizon.[b] A star in the equator

[a] A common celestial globe, or even a reference to the concave surface itself, will much better assist the conception of the circles of the sphere, than figures drawn on a plane surface, which are rather apt to mislead a beginner. The horizon of the globe must be considered as continued to pass through the centre, where the eye is supposed situate viewing the hemisphere above the horizon, and the axis of the globe is to be placed at the same elevation as the axis of

is only visible while in the part EUW (equal to WQE). A star in XLF is only visible in the portion XLF above the horizon; it rises at X, and sets at F. ZTK is a portion of a secondary to the horizon. TK is the altitude of the point T, and TZ its zenith distance. PTD is a secondary to the equator, or a circle of declination, and DT the declination of the point T.

A telescope being directed to any star, and the time noted by a clock, if the telescope remain fixed, the same star will again pass through it after an interval of 23^h 56^m nearly. And the time of passing over the aperture of the telescope being the same to whatever part of the star's diurnal path the telescope is directed, proves the equable motion in that diurnal path. A telescope particularly fitted up, and placed so as to be conveniently moved in the plane of the meridian, is of as much use in the practice of astronomy as the quadrant: it is called a transit instrument; its uses will be afterwards explained, as well as the method of finding the direction of the meridian.

The time of describing a diurnal circle by a star may be nearly ascertained, without a telescope, by suspending two plumb lines at two or three feet from each other, then observing when the star appears in the plane of the strings, noting the time by a clock well regulated: the same star will pass the plane again after 23^h 56^m. An upright wall will serve for the same purpose. *Vice versâ*, this method will serve to ascertain the rate of going of a clock. It may also be applied to ascertain the time of passage over the meridian, by adjusting the plumb lines in the plane of the meridian.

Secondaries of the equator are also called *hour circles*, because the arc of the equator, contained between any

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the concave surface of the spectator. In this way all the circles of the celestial sphere will be easily understood. Any consideration of the form of the earth is entirely foreign to a knowledge of the circles of the sphere. They were originally invented without any reference to or knowledge of it.

[b] Such a star is called a *circumpolar* star.

one of these circles and the meridian, shows the distance in time of that body from the meridian, the equator being divided into 24 hours.

9. The meridian also passes through the *nadir* (the lower pole of the horizon).

Secondaries of the horizon are called *vertical circles.* That vertical circle which intersects the meridian at right angles is called the *prime vertical.*

It will help the conception of the student to consider the meridian and other verticals of the horizon as remaining at rest, while the sphere revolves, carrying with it the equator and other circles.

The four points where the meridian and prime vertical intersect the horizon, are called the cardinal points; those of the meridian, north and south; those of the prime vertical, east and west. The equator intersects the horizon in the east and west points (being poles of the meridian), and its inclination to the horizon equals the complement of the altitude of the celestial pole; for the inclination at E of the great circles ES and EU is measured by the arc SU, which is equal to PZ, since they have the same complement UZ. The prime vertical also intersects the equator at the east and west points, and at an angle equal to the altitude of the pole, since, if the arc EZ be drawn, the angle ZEU is measured by UZ, which is equal to PN, both having the same complement PZ.

The *azimuth*[a] of a celestial object is measured by an arc of the horizon, intercepted between the meridian and a vertical circle passing through the object. In (Fig. 2) KN is the azimuth of the point T from the north.

The altitude of a celestial object T, being its distance from the horizon measured on a secondary of the horizon, is greatest when the object is on the meridian, for its zenith distance ZT is then least, for PZ + ZT is

[a] The complement of the azimuth, or the arc intercepted between the prime vertical and the vertical through the object, is called the *amplitude.*

greater than PT (=PY), consequently ZT is greater than ZY ; Y denoting the position of the star when on the meridian.

9. The apparent path of the sun traced on the surface of the celestial sphere, among the fixed stars, is a great circle, which he moves over, in a direction from west to east. This circle is called the *ecliptic*, because eclipses take place when the moon, at the new and full, is in or near this circle. The apparent motion of the sun, in this circle, is not entirely uniform; the motion being contrary to the *diurnal* motion, the interval between two meridian passages of the sun is greater than that of the fixed stars by four minutes nearly. This interval, between two passages of the sun over the meridian, is in its mean quantity called 24 hours, or a day. In 365 days, 6 hours, and 9 minutes, the sun appears to complete the ecliptic. The seasons are connected with the positions of the sun in the ecliptic. The period, therefore, of his motion, called a year, becomes one of the most important divisions of time.

10. The moon completes her course among the fixed stars, by a motion from west to east, in 27 days 7 hours, returning nearly to the same place. Its apparent path is *nearly* a great circle, intersecting the ecliptic at an angle of about five degrees. Its motion also being contrary to the diurnal motion, the interval between its successive passages or transits over the meridian is greater than that of the fixed stars by 52 minutes, in its mean quantity. The moon is said to be in opposition to the sun, when near that part of the ecliptic opposite to the sun. The interval between two oppositions is nearly 30 days, and at each opposition the moon shines with a full phase. The use, in civil life, of this striking phenomenon, makes another important division of time, which is called a *month.*

11. The ecliptic necessarily intersects the equator, each being a great circle. The angle of intersection is nearly 23° 28′. The circumstance of the inclination, or *obliquity of the ecliptic* to the equator, explains the change of seasons. The true cause of the appearance

of the obliquity of the ecliptic to the equator, will be afterwards shown. If the ecliptic coincided with the equator, the sun would always rise and set in the east and west points, would always be at the same altitude when on the meridian, and would be absent and present during equal spaces of time. Now the effect of the sun, with respect to heat, depends upon the time of his continuance above the horizon, and the greatest altitude to which he rises; therefore, if he moved in the equator, no alteration would take place, because these would be the same every day. But the ecliptic being inclined to the equator, when the sun is in that part which is between our visible pole and the equator, the greater part of each of the diurnal circles which he describes, is above our horizon, i. e. he is more than half the 24 hours above the horizon, and he passes the meridian between the equator and zenith. When southward of the equator, he is less than 12 hours above the horizon. When he is in the points of intersection of the ecliptic and equator, he is just 12 hours above the horizon, and it is then equal day and night. This latter circumstance takes place on the 21st of March and 23rd of September.

The sun is in that part of the ecliptic nearest our visible pole about the 21st of June, and then our days are longest, and in the part farthest from it on the 21st of December, when our days are shortest. The sun is about eight days longer on the northern side of the ecliptic than on the southern, and hence summer is eight days longer than winter.[a] The greatest heat is not when the days are longest, but some time after, because the increase of the heat during the day is greater than the decrease during the night, consequently heat must accumulate till the increments and

[a] This arises from the fact (to be afterwards explained) that the apparent diurnal motion of the sun in the ecliptic is slower in summer than in winter, owing to the distance of the sun from the earth being then greater.

decrements are equal; afterwards the decrease being greater than the increase, the heat will diminish.

12. The two parallels to the equator, or parallels of declination, touching the ecliptic, are called *tropics* or *tropical circles*, because when the sun is in these points of the ecliptic, he turns (τρέπει) his course, as it were, back again towards the equator.

The points of the ecliptic of greatest declination, or the tropical points, are called *solstices*,[a] because the sun appears stationary, with respect to his approach to the poles.

13. A belt or zone extending on each side of the ecliptic about 8° is called the *zodiac*, from certain imaginary forms of animals conceived to be drawn in it, called *signs* of the zodiac. There are twelve signs, probably from there being twelve lunations during the course of the sun in the ecliptic. These are Aries, Taurus, Gemini, Cancer, Leo, Virgo, Libra, Scorpio, Sagittarius, Capricornus, Aquarius, and Pisces, and denoted by ♈, ♉, ♊, ♋, ♌, ♍, ♎, ♏, ♐, ♑, ♒, ♓. The reason of distinguishing this space was, because the sun and planets were always observed within it. These figures served also to distinguish the position of the stars with respect to one another, and were therefore called the *constellations* of the zodiac. The space of the zodiac has always been noticed from the earliest records of astronomy. Some of the Asteroids are not confined to this space. One of them, Pallas, sometimes is distant above 62° from the ecliptic, as seen from the earth.

The first six constellations, beginning with Aries, were on the northern side of the ecliptic, when the description of the zodiac was first invented, and the six others on the southern. But by a comparison of observations made at a considerable interval from each other, it is found that the intersections of the ecliptic and equator move backward, in respect to

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[a] Derived from the Latin words *sol* and *stare*.

the signs of the zodiac, the obliquity of the ecliptic remaining nearly the same. The equator moves on the ecliptic, the ecliptic continuing to pass nearly through the same stars. The intersections or the equinoctial points move backward at the rate of $50{\cdot}1''$ in a year, or $1°$ in $71\frac{1}{2}$ years, and therefore, at present, the constellation Aries seems to be moved forward nearly $30°$ from the equinoctial point, yet astronomers still commence the twelve signs or divisions of the ecliptic at the equinoctial point, and name them after the constellations of the zodiac. This distinction ought to be attended to.

14. In the practice of astronomy, the most general and convenient method of ascertaining the position of any celestial object on the concave surface, is to determine its position with respect to the equator and vernal equinoctial point, that is, to determine its *declination* and *right ascension*. The *right ascension* of a celestial body is the arc of the equator intercepted (reckoning according to the order of the signs), between the vernal equinoctial point, or the first point of Aries, and a secondary to the equator passing through the object. This is expressed both in time and space. Thus, if the arc intercepted be $15°$, the right ascension may be said to be $15°$ or one hour, supposing the equator divided into twenty-four hours. The time of the diurnal revolution of the fixed stars, or of the celestial sphere, is called a *sidereal day*. It is divided into twenty-four sidereal hours. It is equivalent to 23 hours 56 minutes mean solar time. Hence the interval in sidereal time between the passages of two fixed stars over the meridian, is the same as the *difference* of their right ascensions expressed in *time*.

The term, right ascension, originally had a reference to the rising of the celestial bodies. Now its use is much more circumscribed, but much more important, and therefore it might have been better to have adopted another term for expressing the arc intercepted between a secondary to the equator passing through the celestial object, and the first point of Aries.

15. The position of a celestial body, with respect to the equator, being ascertained, it is very often necessary to ascertain its position with respect to the ecliptic, i. e. to determine its *longitude and latitude.* This is done by spherical trigonometry, thus—

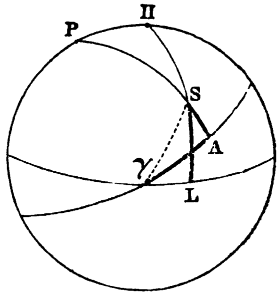

Fig. 3.

Let Π be the pole of the ecliptic, P the pole of the equator, S a star, γ the intersection of the equator and ecliptic. SL will be the star's latitude, γL its longitude, SA the declination, γA the right ascension, draw the arc of a great circle γS; being given γA, AS, we can calculate γS and the angle SγA; also the obliquity of the ecliptic AγL is known; hence in the spherical triangle SγL we have Sγ, and SγL, and therefore SL, γL can be found.

The *longitude* of a celestial object is measured by an arc of the ecliptic, intercepted between the first point of Aries (reckoning according to the order of the signs of the zodiac) and a secondary to the ecliptic passing through the object. Its *latitude* is its distance from the ecliptic, measured on a secondary of the ecliptic passing through the object.

16. The *solstitial colure* is a secondary to the equator passing through the solstices, and is therefore also a secondary to the ecliptic.

The *equinoctial colure* is a secondary to the equator, passing through the equinoctial points.

CHAPTER II.

FIXED STARS—TELESCOPES—APPEARANCE OF STARS IN TELESCOPES.

17. Let us now return to the consideration of the fixed stars. We observe about 3000 stars visible to the naked eye, very irregularly scattered over the concave surface of the heavens. There are seldom above 2000 visible *at once*, even on the most starry night. They are distinguished from the planets not only by preserving the same relative position to each other, but also by a tremulous motion or twinkling in their light, apparently arising from the effect of the atmosphere on the rays of light passing through it.

For the conveniency of arranging and referring to the different stars, the method of constellations was invented by the ancients. They imagined a number of personages of their mythology, also animals, &c., drawn on the concave surface, and including particular groups of stars; these they called constellations, and denominated the stars from the constellation in which they were, and from their situation in that constellation. This method, though certainly useful, is not adequate to the purposes of astronomy in its present state, but for many obvious reasons it has been retained. The stars do not form the figure of the constellation, except in a few assemblages which have a remote resemblance; such are the Great Bear, the Hyades composing the Bull's head, &c. Some of the brighter fixed stars, and those more remarkable by their position, had proper names assigned to them, as Arcturus, Sirius, Alioth, Algol, &c.

18. MAGNITUDES OF STARS.—Bayer, who published a celestial atlas in the year 1603, much facilitated the arrangement of the fixed stars, by marking those in each constellation by the letters of the Greek alphabet; their order in the alphabet has, however, no exact relation to the brightness of the stars, as was formerly supposed to be the case. The stars are also divided according to their apparent brightness into magnitudes. The brightest are of the first magnitude, and so on to the sixth, the least magnitude visible to the naked eye. There are thirteen stars of the first magnitude in the portion of the concave surface visible to us, viz.:—Aldebaran (α Tauri); Capella (α Aurigæ); Rigel (β Orionis), (α Orionis); Sirius (α Canis Majoris); Procyon (α Canis Minoris); Regulus (α Leonis); Spica Virginis (α Virginis); Arcturus (α Boötis); Antares (α Scorpii); α Lyræ; α Aquilæ; and Faumalhaut (α Piscis Austral.). In the remaining portion of the concave surface there are six, viz.:—Achernar (α Eridani); Canopus (α Argus); Argûs (variable); α Crucis, α and β Centauri. There are about 50 of the second magnitude, and about 120 of the third magnitude, visible to us.

By the assistance of telescopes we find that the number of the fixed stars is greater than can be ascertained; those which are visible to the naked eye being incomparably fewer in number than those which are visible by the aid of telescopes. The naked eye remarks also a very considerable luminous belt stretching over the heavens, almost in a great circle, which is called the Milky Way. By the aid of powerful telescopes this is found to consist entirely of small stars. There are also other small luminous spots—one, for instance, in the constellation Cancer, and one in Perseus, which even small telescopes will resolve into a mass of small stars.

The group in Taurus, called the Pleiades, in which six or seven stars may be noticed even with the naked eye, is found, when examined by the telescope, to consist of between 50 and 60 stars; and the constellation Coma Berenices is another such group, in which,

however, the stars are more scattered. Besides these the telescope reveals a number of small luminous and nebulous-looking spots called Nebulæ. Some of these are resolvable by powerful telescopes into a mass of densely packed stars, and are called Clusters.

19. Telescopes.—The theory of telescopes properly belongs to the science of optics, and therefore a very short account of the effects which they produce, and of the improvements that have, from time to time, been made in them, is all that is necessary here.

The use of telescopes is to magnify objects, or to present their images under a larger angle than the objects themselves subtend; and likewise to render objects visible that would otherwise be invisible. Telescopes for common astronomical purposes magnify from 40 to 200 times, and for particular purposes from 200 to 1000 and upwards; i. e. objects appear so much nearer than when seen by the naked eye, and their parts become more visible and distinct. We are enabled by a telescope which magnifies 100 times to behold the moon the same as we should if placed 100 times nearer than at present. A telescope magnifying a thousand times will exhibit the moon as we should behold it could we approach within 240 miles of it. Thus, although we cannot actually approach the moon at pleasure, we can form an image of the moon, and approach this image at pleasure, and so make the image subtend a greater angle than the moon itself. We can magnify the image by help of a simple microscope, as we can magnify any minute object. This is the principle of the common telescope. The object glass forms an image of the moon, and we magnify this image by help of the eye glass, which may be considered as a microscope.

20. Telescopes were accidentally invented at Middleburgh, in Holland, about the year 1609. There is no foundation for supposing them known earlier, although the single lens had been in use for spectacle glasses since the beginning of the 14th century. Galileo, hearing of their effects, soon discovered their con-

struction, and applied them to astronomical purposes, from whence a new æra may be dated in astronomy. After some trials, Galileo made a telescope which magnified upward of 30 times; and with this instrument, so inferior in power to modern telescopes, he made most important discoveries. In little more than a year he had observed the nebula of Orion, the telescopic stars in the Pleiades and in Præsepe; had discovered the satellites of Jupiter, very accurately described the face of the moon, and computed the height of some lunar mountains, observed an extraordinary appearance in Saturn, occasioned by the ring, which his telescope could not clearly show, and had observed phases in Venus similar to the phases of the moon.

Notwithstanding the importance of the telescope, it was but slowly improved. Telescopes admitting of a high magnifying power were of a very inconvenient length. A high magnifying power could not be obtained by a short telescope, without rendering the image indistinct by colour. The ardour and industry of the astronomers of the latter end of the 17th century overcame this difficulty, by using telescopes without tubes. They affixed the object glass to the top of a pole, directing it by means of a long string, so as to throw the image into its proper place. Huyghens particularly distinguished himself by important discoveries with this inconvenient kind of telescope, which has been called the *aerial telescope.* The discoveries of Sir Isaac Newton respecting light induced astronomers to desist from endeavouring to improve refracting telescopes, and to aim at perfecting reflecting ones. Soon after the discovery of the telescope, it was suggested that the image of the object might be formed by reflection, instead of refraction; but, as no particular advantage could be shown to arise from this alteration, it does not seem to have been attended to, till James Gregory proposed the construction of a reflecting telescope, which goes by his name. He intended by this construction to obviate the errors of the object glasses of the common telescope, arising from their

being necessarily ground of a spherical form. The discoveries of Newton on light showed these errors to be comparatively of trifling consequence. Newton himself, as soon as his experiments on light had shown him the true obstacle to the improvement of refracting telescopes, invented and executed a reflecting telescope, which goes by his name. His construction is better adapted to many purposes in astronomy than that of Gregory, although for common purposes Gregory's may be considered most convenient.

Many inconveniences attended the construction and execution of reflecting telescopes. When made, they were liable to tarnish, and to change their figure, an error in which is of much greater consequence than in refractors. Thus much fewer advantages were derived from reflecting telescopes than had been expected. And the improvement of telescopes seemed at a stand, when, in the year 1757, a discovery of Mr. Dolland, an optician in London, gave hopes of improving them far beyond what had been hitherto done. He discovered that by a combination of lenses of flint glass and crown glass, he could form an image free from colour. This enabled him to make telescopes, admitting of high magnifying powers, of a very convenient length. These telescopes, called *achromatic*, are now in common use, and fitted to those astronomical instruments by which angles are measured. They were at first of small dimensions, owing to the difficulty of obtaining large homogeneous discs of flint glass. Fraunhofer, of Munich, first succeeded in making them of large size; his first large instrument being the celebrated 9-inch refractor, made for the Dorpat Observatory. Still larger ones were afterwards made by his successors—Merz; Couchoix, in Paris; Clark, in America; Cooke, in York; and others. The largest refractor now in use is that made by Clark for the Observatory of Chicago, the diameter of the object glass of which is 18 inches. Another has lately been finished by Mr. Cooke of 21 inches diameter.

21. The reflecting telescopes were first greatly improved by the elder Herschel. After repeated attempts

he succeeded in making one 20 feet long, and 18 inches aperture. The great breadth of the aperture increased so much the brightness of the image, that he was enabled, with great convenience, to use very high magnifying powers. At last he attempted and executed one 40 feet in length, and of 4 feet aperture. In order to form some idea of the effect of telescopes, when applied to the celestial bodies, it may be remarked that the reflector of the 40 feet telescope forms an image of the ring of Saturn, about $\frac{1}{10}$th of an inch in diameter; we are enabled to magnify this by the eye glass, in the same manner as we can magnify an object $\frac{1}{10}$th of an inch in breadth by a common microscope. This telescope has been only surpassed in size by the large reflector of 6 feet diameter, and 52 feet focal length, constructed by the late Earl of Rosse, which is the most powerful instrument in existence. One of the same size as Herschel's was constructed by Mr. Grubb, of Dublin, for the Melbourne Observatory.

22. The appearance of the stars seen in a telescope is very different from that of the planets. The latter are magnified, and show a visible disc. The stars appear with an increased lustre; but with no disc. Some of the brighter fixed stars appear most beautiful objects, from the vivid light they exhibit. Sir W. Herschel tells us, that the brightness of the fixed stars of the first magnitude, when seen in his largest telescope, is too great for the eye to bear. When the star Sirius was about to enter the telescope, the light was equal to that on the approach of sunrise, and upon entering the telescope, the star appeared in all the splendour of the rising sun, so that it was impossible to behold it without pain to the eye.

The apparent diameter of a fixed star is only a deception arising from the imperfections of the telescope. The brighter stars appear sometimes in bad telescopes to subtend an angle of several seconds, and this has led astronomers into mistakes respecting their apparent diameters. The more perfect the telescope, the less this irradiation of light. We know certainly that some

of the brighter fixed stars do not subtend an angle of 1″, from the circumstance of their instantly disappearing, on the approach of the dark edge of the moon.

23. Although the superior light of the sun effaces that of the stars, yet by the assistance of telescopes we can observe the brightest stars at any time of the day. The aperture of the telescope collects the light of the star, so that the light received by the eye is greater than when the eye is unassisted. The darkness in the tube of the telescope also in some measure assists.[a]

The most inferior telescope will discover stars that escape the unassisted sight. By the telescope we discover that the Milky Way, and some of the nebulæ above mentioned, consist of very numerous small stars. Others, even in the best telescopes, appear still as small luminous clouds. There is a very remarkable one in the constellation of Orion, which the best telescopes show as a spot uniformly bright. It is a singular and beautiful phenomenon. So great is the number of telescopic stars in some parts of the Milky Way, that Sir W. Herschel observed 588 stars in his telescope at the same time, and they continued equally numerous for a quarter of an hour. In a space about 10 degrees long, and 2½ degrees wide, he computed there were 258,000 stars. "Phil. Trans." 1795.

24. Catalogues.—The most ancient catalogue of the fixed stars is that of Hipparchus, who observed at Alex-

[a] It appears by the principles of optics, that when an object is seen through a telescope, the density of the light on the retina must be always less than when the object is seen by the naked eye; but the quantity of light in the whole image may be much greater in the former case than in the latter. And it is certain that our power of seeing the object with distinctness depends on the quantity of light in the whole image. Sir W. Herschel, in a valnable paper in the "Phil. Trans." 1800, Part I., on the power of penetrating into space, uses the terms *absolute brightness* and *intrinsic brightness*—the former to distinguish the whole quantity of light in the image on the retina, and the latter to distinguish its density. He gives an instance in which the absolute brightness was increased 1500 times in a telescope, and the intrinsic brightness was less than to the naked eye in the proportion of 3 to 7.

andria, about 150 B. C. His catalogue consists of 1022 stars. Although several celebrated astronomers, as Tycho Brahe, &c., employed themselves in more accurately observing the places of the fixed stars, yet the number was not much increased till the time of Flamstead, whose catalogue, entitled the British Catalogue, appeared in 1725, and contains about 3000 stars visible to the naked eye. The observation of the places of these stars has been since that time the chief business of the principal observatories, specially that of Greenwich, and they are now known to an extraordinary degree of accuracy. In 1802, M. Delalande published a work entitled *Histoire céleste Française,* in which are observations of 5,000 stars, viz., of stars of the 6th magnitude not observed by Flamstead, and of telescopic stars of the 7th, 8th, and 9th magnitudes. They were mostly observed by his nephew, M. Lefrançais Delalande. These have been followed by the observations of stars in zones made by Bessel, and continued by Argelander, and at present a number of observatories are engaged in re-observing all stars in the northern hemisphere almost down to the 10th magnitude, which had been previously mapped and catalogued by Argelander, and whose number exceeds 300,000.

25. Double Stars.—Some stars, appearing single to the naked eye when examined with a telescope appear double or treble—that is, consisting of two or three stars very close together: such are Castor, *a* Herculis, the Pole Star, &c. Seven hundred, not noticed before, have been observed by Sir W. Herschel; and since that time they have been the special object of some observers as Sir John Herschel, and especially the elder Struve, who, in his great work, "*Mensuræ micrometricæ Stellarum Duplicium,*" gave the accurate measurement of more than 3000 double and triple stars. Some of these stars show a regular motion round each other, and are called binary stars. In viewing these double stars a singular phenomenon discovers itself, first noticed by Sir W. Herschel—some of the double stars are of different colours, which, as the images are so near each other,

cannot arise from any defect in the telescope. *α* Herculis is double, the larger red, the smaller blue; *ε* Lyræ is composed of four stars, three white, and one red; *γ* Andromedæ is double, the larger reddish-white, the smaller a fine sky blue. Some single stars evidently differ in their colour. Aldebaran is red, Sirius brilliant white. These double stars form also very good objects for testing the goodness of telescopes, for if the instrument do not give a well-defined image those stars will appear as one.

26. New Stars.—From observations at different periods, it appears considerable changes have taken place among the fixed stars. Stars have disappeared, and new ones have appeared. The most remarkable new star recorded in history was that which appeared in 1572, in the Chair of Cassiopæa. It was for a time brighter than Venus, and then seen at mid-day: it gradually diminished in lustre, and after sixteen months disappeared. Cornelius Gemma viewed that part of the heavens on November 8, 1572, the sky being very clear, and saw it not. The next night it appeared with a splendour exceeding all the fixed stars, and scarcely less bright than Venus. Its colour was at first white and splendid, afterwards yellow; and in March, 1573, red and fiery like Mars or Aldebaran; in May of a pale livid colour, and then became fainter and fainter till it vanished.

Another new star, little less remarkable, appeared in October, 1604. It exceeded every fixed star in brightness, and even appeared larger than Jupiter. Kepler wrote a dissertation upon it.

In our own time, on the 12th of May, 1866, a new star was suddenly observed in the constellation Corona, as bright as *α* Coronæ or Gemma; that is, of the second magnitude. This star, after its place had been carefully determined, was found to be not a new star, but a fixed star, which was put down in Argelander's Catalogue as between the 9th and 10th magnitude, and which had suddenly thus increased in brightness. The star diminished very rapidly in a few days in bright-

ness, and by the end of June was again of its usual magnitude (9·5). Since then it has remained so with only very slight variations. The observations made on this star during his short bright period, have enabled astronomers to discover the cause of this sudden outburst of light, which undoubtedly was the same in the case of the stars mentioned before. We shall refer to these observations herafter.

Changes have also taken place in the lustre of the permanent stars; β Aquilæ is now considerably less bright than γ. A small star near ζ Ursæ Majoris is now probably more bright than formerly, from the circumstance of its being named Alcor, an Arabic word which imports sharp-sightedness in the person who could see it. It is now very visible.

27. Periodical Stars.—Several stars also change their lustre periodically; o Ceti, in a period of 333 days, varies from the 2nd to the 6th magnitude. The most striking of all is Algol or β Persei. Mr. Goodricke has with great care determined its periodical variations. It is, when brightest, of the 2nd, and when least, of the 4th magnitude; its period is only $2^d\ 21^h$: it changes from the second to the fourth magnitude in $3\frac{1}{2}$ hours, and back again in the same time, and so remains for the rest of the $2^d\ 21^h$. Besides these, the most remarkable periodical stars are—β Lyræ, which has a period of $12^d\ 22^h$, in which a double maximum and minimum occurs, the two maxima being nearly equal, while the minima are exceedingly unequal; δ Cygni, η Argûs in the southern hemisphere, &c. Argelander, of Bonn, has devoted special attention to this subject; and it is chiefly owing to his discoveries, and those of his pupils, that the number of stars known to be periodical has been greatly increased.

28. Nebulæ.—The number of nebulæ is very remarkable. The number known before Sir W. Herschel's time was only 103, contained in Messier's Catalogue; but he discovered with his large telescope about 2000. Their number was still farther increased by Sir John Herschel, who went to the Cape of Good Hope to observe

the nebulæ and double stars in the southern hemisphere, and by others; and the number of nebulæ given in Sir John Herschel's Catalogue, extending over the entire hemisphere, is above 4000. The great majority of these can be seen only through powerful telescopes. These nebulæ are not evenly distributed, the hours, 3, 4, 5, and 16, 17, 18, of right ascension in the northern hemisphere being very poor, while the hours 10, 11, 12, specially the last, are exceedingly rich in them. The apparent forms and the character of these nebulæ are very varied. We may, however, divide them into several distinct classes:—1°. Clusters, in which the component stars are clearly distinguishable, and which are either globular and dense, as the cluster between η and ζ Herculis, or irregular and less defined in outline, as the one surrounding the star κ Crucis, consisting of about 110 stars from the 7th magnitude downwards, some of which display beautifully rich colours. 2°. Resolvable nebulæ, or such as excite suspicion that they consist of stars, and may be resolved with more powerful telescopes. 3°. Nebulæ, properly so called, in which there is no appearance whatsoever of stars. As all the globular clusters appear in small telescopes like nebulæ of the second class, so also many which hitherto have been considered to be real nebulæ would belong to the second class, if viewed through more powerful telescopes. Lord Rosse's telescope has resolved many which had resisted all inferior instruments. 4°. Planetary nebulæ, so called from their perfect resemblance to planetary discs, without any central condensation. These are round or oval, in some instances sharply defined, in others a little hazy at the borders. The largest of these is in the Great Bear, having an apparent diameter of 2′ 40″. 5°. Nebulous stars, which are stars surrounded by a nebula which is often perfectly round, the star standing in the centre.

The forms of the nebulæ of the second and third classes are exceedingly varied. Besides round nebulæ condensed towards the centre, which probably are only

distant globular clusters, and which in some instances are even double, the perfect counterpart of a double star, we have found them elliptical, more or less elongated. The best specimen of this kind is in the girdle of Andromeda, which is visible to the naked eye. Among the annular nebulæ the most remarkable is that half-way between β and γ Lyræ, which can be seen with telescopes of moderate powers; but is resolved into a mass of minute clustering stars by Lord Rosse's telescope. The spiral nebulæ are a very remarkable class, discovered first by Lord Rosse. The most remarkable of these is that which was first discovered (51 in Messier's Catalogue). It had been described by Sir W. Herschel as a globular nebula, surrounded by a ring at a considerable distance from the globe, which is subdivided into two laminæ at one part of the circumference. The powerful telescope of Lord Rosse shows it to consist of several convolutions of a spiral starting from a common centre, and a smaller companion connected with the spiral branches.

The most conspicuous of the nebulæ, of great extent, without any symmetry of form, is the great nebula in the sword handle of Orion. The brightest portion of this offers a resemblance to the open jaws of an animal with a kind of proboscis. Many stars are seen scattered over it. The bright star called θ, when seen through a telescope, consists of four bright remarkable stars, forming a trapezium, two of which, however, are close double stars. They occupy a conspicuous situation, close to the brightest portion of the nebula, but no nebula exists within the area of the trapezium.

The brightest portion of the nebula, when seen through Lord Rosse's telescope, has the appearance of consisting of clustering stars. The most perfect map of this nebula was published by the present Lord Rosse. Other remarkable nebulæ of this class are that surrounding η Argûs, in which great changes seem to have taken place since the time of observation by Sir John Herschel, and the Magellanic clouds, or Nubecula Major and Minor, the former of which occupies

an area of 42 square degrees, the latter of ten square degrees. They consist of large patches of nebulous matter, in all stages of resolution, and of irresolvable matter, and besides many regular and irregular clusters and nebulæ, of which 278 may be counted in Nubecula Major alone.

29. Having given a short statement of the simple appearances of these bodies placed on the concave surface of the heavens, which are such, that they must strongly excite our curiosity; we may now leave the subject, and resume it after having stated the knowledge that observations and deductions from thence afford us, respecting the magnitudes, distances, and motions of the sun, moon, and planets. Then returning again to the consideration of the fixed stars, and assigning them their proper places in the universe, we shall discover what must fill our minds with astonishment and awe, and must raise in us the greatest admiration of the Almighty Creator. That which has hitherto been stated regards only what a spectator fixed to one spot might discover. It is only by a change of place, or by comparing the observations made at places distant from each other, that we can readily arrive at a knowledge of the real distances and real motions of the celestial bodies. An isolated observer, however he might be gratified by the spectacle of the heavens on a fine evening, would be able to discover little of that which, when the true circumstances are known, adds so much to the wonderful variety of the Creator's power which we observe in terrestrial matters. He would only barely discover that the sun, moon, and planets were at different distances from the earth. He would also be able to form hypotheses to explain their motions, but few of those would he be enabled to submit to the test of experience. Previously to this it would be necessary to investigate the figure and dimensions of the earth upon which he lives. This knowledge is obtained from the phenomena which arise from a change of place.

CHAPTER III.

PHENOMENA DEPENDING ON A CHANGE OF PLACE, AND ON THE FIGURE OF THE EARTH.

30. A SPECTATOR without changing his situation on the earth, would soon discover that the celestial bodies are not all placed on the concave surface at *fixed* distances from him; for he would remark that the sun, moon, and planets varied their apparent magnitudes or diameters, which must arise either from changes of distance, or changes in the actual magnitudes of the bodies. The former solution is so much simpler than the latter, that no one could hesitate in adopting it, even if not confirmed by other circumstances. Likewise that the heavenly bodies are not placed at *equal* distances from him. It was remarked that the apparent paths of the sun and moon intersected each other. When they appear to meet at these intersections, the moon is observed to obscure or eclipse the sun, consequently the moon must be nearer than the sun. But to proceed in the investigation of these distances, it will, as was observed, be necessary to become acquainted with the form of the earth on which we live.

31. A spectator placed on the sea, or on a plain, where his view is unobstructed, at first considers the surface as a plane coinciding with his horizon, and extended to the concave surface of the celestial sphere. But it is immediately suggested to him, that the surface of the earth is not flat or coincident with his horizon, for on the sea he perceives the tops of the masts to disappear last, and on the plain he observes the tops of distant objects, when the bottoms are invisible. This

cannot be explained otherwise than by a curvature on the earth's surface. The voyages of modern navigators have put this matter in the clearest light; for, by continued sailing to the eastward or westward, they have arrived again at the port from which they set out. This has been done in different courses on the surface, so that thereby traversing the earth, they have ascertained its surface to be a curved surface returning into itself. Eclipses of the moon serve to point out that the figure of the earth must be nearly spherical, for the boundary of the earth's shadow seen on the moon always appears circular, which could not *always* be the case, unless the earth were nearly a sphere. Again, the tops of mountains are seen from a distance when their bases are invisible, and the sun can be seen by a spectator at a great altitude, after he has set to the inhabitants of a place at the level of the sea.

32. The magnitude of the earth is next to be considered; previously to which it is necessary to remark, that however distant two places on the earth's surface are, the angular distances of the same stars visible in each place are precisely the same; from whence it follows, that the distances of the fixed stars are so great, that each inhabitant of the earth, in respect to them, considers himself in the centre of the same imaginary sphere; or that all lines drawn from the different points of the surface of the earth to any star, may be considered as parallel; for the inclination of the lines drawn from any two places towards the same star, is less than can be measured, and therefore for all purposes they must be considered as parallel.

33. Every spectator also observes the same celestial pole and equator, that is, situate the same with respect to the fixed stars; but the situation of the celestial circles with respect to the horizon will be different. The meridian altitudes of the celestial objects will be different in different places, and the altitude of the north celestial pole will be increased or diminished, according as an observer travels north or south.

Actual measurement shows, that the space gone over in a direction north or south, is very nearly proportional to the variation of the altitude of the celestial pole. Measurements showing this have been made in Lapland, England, Germany, France, Italy, at the Cape of Good Hope, Hindostan, and in North and South America.

34. From hence it is proved that the earth is nearly a sphere, by which is explained the phenomenon of the variation of altitude of the pole, being proportional to the space gone over in a direction north or south.

THEOREM.—*If the earth be a sphere, and consequently the meridian a circle, the change in the altitude of the pole will be proportional to the space gone over north or south.* For let the circle LCS (Fig. 4) represent a circle of the earth, on the plane of a celestial meridian, LR a section of the horizon of the place L, SO of the place S. LP and SP′ lines drawn in the direction of the celestial pole, which are therefore parallel (Art. 32 and 33). Produce OS to meet LR and LP, in H and B, therefore P′SO (= PBH = BLH + BHL) = BLH + LCS. (Since the quadrilateral LHCS is circumscribable by a circle.) But LCS varies as LS, consequently the difference of the elevations of the pole at L and S varies as LS. Experiment showing this to be nearly so, it follows that the earth is nearly a sphere, since it may be shown by the Integral Calculus that no other curved surface possesses this property. It is also proved by navigators, in distant voyages, making their computations of the distances sailed, upon the supposition that the earth is a sphere, the results of which nearly agree with the distances ascertained by the rate of sailing deduced by the log line.

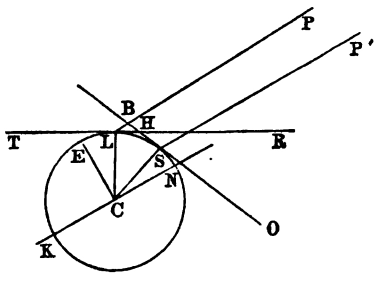

Fig. 4.

35. The measure of a degree on the earth's surface is[a] $69\frac{1}{2}$ British miles nearly, that is, if the difference of PLR and P'SO be 1°, the distance LS = $69\frac{1}{2}$ miles, and therefore 360° or the circumference of the earth = 25000 miles nearly (24,857·5 miles exactly). Hence the diameter, which is somewhat less than $\frac{1}{3}$ of the circumference = 8000 miles nearly (the equatorial diameter is exactly 7,926·708 miles). A vast magnitude, when measured by our ideas, but almost nothing when compared with other bodies, the existence of which in the universe we are enabled to ascertain.

36. It cannot now be determined how long the knowledge of the spherical figure of the earth has existed, but just ideas of it were early entertained. Above 2000 years ago it was commonly known among astronomers. Indeed it must have been discovered in the very infancy of astronomy. It plainly appeared that the eclipses of the moon were occasioned by the intervention of the earth, and the termination of the shadow must soon have pointed out to them the form of the earth. The measure given by Aristotle is the earliest upon record, who reports it from more ancient authors. Eratosthenes, who observed at Alexandria, and died 194 B. C., made use of a method for measuring the earth susceptible of great accuracy.[b] The result

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[a] The method of measuring a degree is afterwards explained in the application of astronomy to geography, by which it is found that the earth is not exactly a sphere, but an ellipsoid of revolution, its equatorial diameter being nearly 27 miles longer than the polar, according to the latest results. The equatorial diameter being 7926·708 miles, and the polar diameter 7899·755 miles, the difference being 26·953 miles, and the ratio of this difference to the equatorial diameter or ellipticity of the earth, is $\frac{1}{291}$. A degree at the equator measures 68·702 miles, and at the pole 69·396.

[b] He knew that at Syene (now Assouan) the sun was vertical at noon on the summer solstice; while at Alexandria at the same moment it was below the zenith by the fiftieth part of the circumference of a circle (7° 42'). The two places differ in longitude by only 2°. Assuming that the sun is so distant that lines drawn to it from the centre and the surface of the earth are parallel, the distance from Syene to Alexandria (5000 stadia) would be the fiftieth part of the circumference of the earth.

of his measurement has come down to us; but from the uncertainty of the length of the stadium used, it has been supposed that we are unable now to appreciate the accuracy of the ancient measurements.

Although the spherical figure of the earth was universally acknowledged among the astronomers, yet the existence of antipodes was long denied.

36. That diameter of the earth, which is parallel to the imaginary celestial axis, is called the axis of the earth, and this is properly so called, because, as will be shown, the earth actually turns upon this axis, thereby causing the apparent diurnal motion of the concave surface.

The great circle of the earth, the plane of which is perpendicular to its axis, is called the *terrestrial equator*, If the plane of this circle be produced to cut the heavens, it will intersect the celestial sphere in the *celestial equator*. Circles are also conceived to be drawn on the earth, corresponding to the imaginary circles in the heavens. The secondary of the terrestrial equator passing through any place, is called *terrestrial meridian* of that place. The arc of the meridian intercepted between the place and the equator, is called the *latitude* of the place, and the arc of the equator intercepted between the meridian of any place and some one given meridian, is called the *longitude* of that place, and is reckoned 180° to the eastward or westward.

37. The British reckon their longitudes from the Observatory of Greenwich; the French from Paris, &c. When the Canary Islands were the most westerly lands known, the longitude was reckoned from the meridian of Ferro, one of those islands. The use of the latitude and longitude in fixing the position of a place on the surface of the earth, was introduced by Hipparchus.

It may be remarked here that the progress of astronomy was from the celestial circles to terrestrial, and not the contrary.

38. By passing to the southward of the terrestrial equator, we are enabled to behold the part of the celestial sphere near the south pole, which is invisible

to us the inhabitants of the northern hemisphere. The stars near the south pole have been divided into constellations. Dr. Halley, De La Caille, and Sir J. Herschel, went to the Cape of Good Hope, for the express purpose of observing the southern hemisphere.

39. The knowledge of the spherical figure of the earth enables us readily to determine the position of the circles of the sphere, with respect to the horizon of any place, the latitude of which is known. For,

The altitude of the celestial pole at any place, is equal to the latitude of that place.

Let NSLE and TLH (Fig. 4, page 31) be sections of the earth and horizon, in the plane of the meridian of the place L. LP the direction of the celestial pole, parallel to the axis CN. CE the terrestrial equator. Then PLC = KCL, and therefore taking from each a right angle, PLR = ECL, the latitude of the place L. Art. 36.

40. Hence it will be easy to understand the changes of seasons over the whole earth. But it is necessary to premise that all observers, who observe the sun at the same instant, refer it nearly to the same place in the celestial sphere. It will be shown hereafter that the greatest difference of place is 17″, and therefore we may consider the sun as appearing to describe the same great circle to all the inhabitants of the earth.

The reader should also bear in mind that the celestial sphere, and the earth, are concentric, and that the terrestrial horizon is a tangent plane to the earth's surface at the place of observation, which, when indefinitely extended so as to intersect the heavens, will cut the celestial sphere in the celestial horizon; the radius of the earth is so small compared with the distances of the heavenly bodies, that this tangent plane may be supposed to pass through the common centre of the earth and heavens, and hence the heavenly horizon will be a great circle.

41. In all places having north latitude, the portion of the northern parallels of declination above the horizon will be greater than those below the horizon, and consequently when the sun is on the northern side of the celestial equator, the days will be longer

than the nights, since the sun's apparent diurnal motion is in a small circle parallel to the equator; the portions of the southern circles of declination above the horizon will be less than those below it, and therefore when the sun is on the southern side of the celestial equator, the days will be shorter than the nights. The contrary will take place in southern latitudes.

For all places, except at the equator and poles, the sphere (reference being had to the position of the parallels of declination, with respect to the horizon) is called an *oblique sphere,* and the sun, moon, and stars rise and set at oblique angles to the horizon.

42. At the equator the celestial poles are in the horizon, and hence the celestial equator and parallels of declination are all perpendicular to the horizon, and are bisected by it, and therefore at the equator all the heavenly bodies appear and disappear during equal times, and their motions at rising and setting will be at right angles to the horizon. This position of the sphere is called a *right sphere.*

43. At the terrestrial poles, the celestial poles appear in the zenith, and the celestial equator coincides with the horizon; the parallels of declination are parallels to the horizon. At the north pole the southern parallels of declination are invisible, therefore the sun is there invisible during six months. This position of the sphere is called a *parallel sphere.*

The circumstances mentioned in the three last articles follow from Art. 39. (Fig. 5) will illustrate what has been said of an oblique sphere; (Fig. 6) of a right sphere; and (Fig. 7) of a parallel sphere. In these

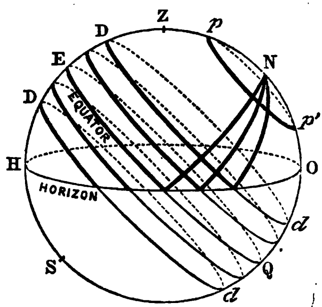

Fig. 5.

figures S and N represent the poles, EQ the equator, HO the horizon, and D*d*, D*d* parallels of declination. The sphere is supposed to be viewed at right angles to the plane of the meridian, the circles of the sphere in planes perpendicular to the meridian being projected into elongated ellipses.

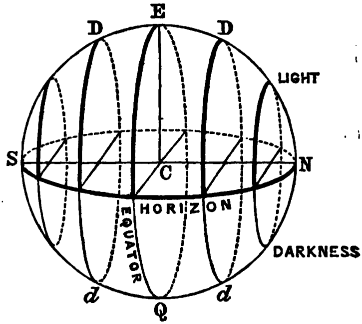

Fig. 6.

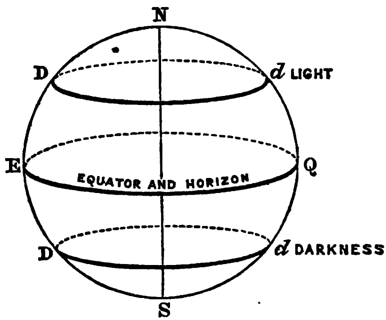

Fig. 7.

44. At places having $66\frac{1}{2}°$ northern latitude, the northern parallel of declination, which is $23\frac{1}{2}°$ from the equator, will just touch the horizon; hence as the sun in his diurnal motion describes this parallel at the summer solstice, the inhabitants of places that have $66\frac{1}{2}°$ north latitude will then observe the sun during 24 hours. The same takes place at the winter solstice for places having $66\frac{1}{2}°$ southern lat.

45. The ancients divided the globe into five principal zones. The zone $23\frac{1}{2}°$ and $66\frac{1}{2}°$ on each side of the equator, is called the *torrid zone.* The sun is always vertical to some place in this zone. The two zones between lat. $23\frac{1}{2}°$ and $66\frac{1}{2}°$ are called *temperate zones;* the two zones about the poles are called the

frigid zones. The parallel of latitude bounding the northern frigid zone is called the *arctic circle,* and that bounding the southern, the *antarctic.*

The parallel separating the torrid zone and northern temperate zone, is called the northern tropical circle; the sun, when in the beginning of Cancer, is vertical to this circle. The parallel separating the southern temperate zone from the torrid zone, is called the southern tropic; the sun when in the beginning of Capricorn is vertical to this.

The ancients also divided the globe into zones, the middle of each zone differing half an hour in the length of their longest day. From the small extent of their knowledge of the surface of the earth, they imagined that places in the same zone, which they called *climate,* differed little in temperature. If so, many parts of Siberia ought to be of the same temperature as Ireland: hence the propriety of disusing the division of the globe into climates.

CHAPTER IV.

ON REFRACTION AND TWILIGHT.

46. As connected with the earth, we may here consider its atmosphere, and how it affects the apparent places of the heavenly bodies. We know, from the science of Hydrostatics, that the air surrounding the earth is an elastic fluid, the density of which is nearly proportional to the compressing force, or the weight of the incumbent air. Whence it follows that the density *continually* decreases, and at a few miles high becomes very small. Now a ray of light passing out of a rarer medium into a denser, is always bent out of its course toward the perpendicular to the surface, on which the ray is incident. Consequently, since the atmosphere consists of a series of concentric strata of air of continually decreasing density from the earth's surface upwards, it follows that a ray of light must be *continually* bent in its course through the atmosphere, and describe a curve, the tangent to which curve, at the surface of the earth, is the direction in which the celestial object appears. Consequently, since this tangent points upwards from the real place of the star, the apparent altitude is always greater than the true, the azimuth remaining the same.

47. The refraction or deviation is greater, the greater the angle of incidence, and therefore greatest when the object is in the horizon. The horizontal refraction is about 35′. At 45° altitude, in its mean quantity, it is $57\frac{1}{3}''$.

48. The refraction is affected by the variation of the quantity or weight of the superincumbent atmosphere

at a given place, and also by its temperature. In computing the quantity of refraction, the height of the barometer and thermometer must be noted. The quantity of refraction at the same zenith distance varies nearly as the height of the barometer, the temperature remaining constant. The effect of a variation of temperature is to diminish the quantity of refraction about $\frac{1}{480}$th part for every increase of one degree in the height of the thermometer. Therefore, in all accurate observations of altitude or zenith distances, the height of the barometer and thermometer must be attended to.[a]

The *law* of atmospheric refraction is found in this manner to be as follows :—The refraction varies as the tangent of the zenith distance, or is equal to a constant quantity (the coefficient of refraction) multiplied by the tangent of the zenith distance.[b]

For altitudes above 10°, we may consider the refraction to be the same as if it took place at a single surface of a spherical shell of air of uniform density, concentric with the earth (Fig. 8). Let SO be the direction of the ray light coming from the star S, and falling on the external surface of the atmosphere at O, it is bent in the direction OH, and reaches the eye at H. The star is elevated towards the zenith Z, through the angle SOS′ = SOZ − HOC (since HOC = S′OZ).

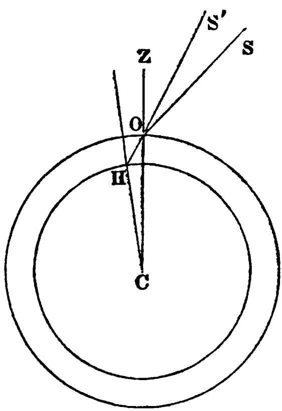

Fig. 8.

[a] Theory shows that, whatever be the law of change of density, the variation of refraction is as the tangent of the zenith distance, between the zenith and about 74° zenith distance. At greater zenith distances we cannot apply theory to obtain the variation of refraction, because there the variation of the density of the air at different heights will

Now, by the laws of optics, sin SOZ $=\mu$ sin COH, (μ being the index of refraction, a constant quantity) $\therefore$ sin (SOS′ + S′OZ) $= \mu$ sin S′OZ $\therefore$ sin SOS′ cos S′OZ + cos SOS′ sin S′OZ $= \mu$ sin S′OZ; but SOS is so small that its cosine $=1$, and its sine nearly equals SOS′ $\therefore$ S′OS. cos SOZ′ + sin S′OZ $= \mu$ sin S′OZ, or SOS′ $= (\mu - 1)$ tan S′OZ.

49. The *coefficient* of refraction may be found by observing the greatest and least altitudes of a circumpolar star. The sum of these observed altitudes diminished by the sum of the refractions corresponding to each altitude, is equal to twice the altitude of the pole, since the star appears to describe a small circle round the pole; then (see Fig. 5, page 35) $Op + Op' = 2ON$, since $Np = Np'$. From whence (if the altitude of the pole be otherwise known), the sum of the refractions will be had; and from the law of variation of refraction, known by theory, the proper refraction to each altitude may be assigned. Thus, if Z and Z′ be the corresponding zenith distances $\{(90° - Z) - K \tan. Z\} + \{(90° - Z') - K \tan Z'\} = 2$ latitude $\therefore$ K (the coefficient of refraction) $= \dfrac{180° - (Z + Z' + 2 \text{ lat.})}{\tan Z + \tan Z'}$.

50. Otherwise, when the height of the pole is not known, the ingenious method of Dr. Bradley may be followed, who observed the zenith distances of the sun at the summer and winter solstices (the sum of which equals twice the latitude), and the zenith distances of

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sensibly affect the quantity of refraction, and the law of this variation is unknown.

[b] That refraction is increased as we approach the horizon, and is proportional to the tangent of the zenith distance, may be proved in a different way, thus:—$\dfrac{\sin\theta}{\sin\phi} = \mu$; ($\theta$ is the angle of incidence, ϕ angle of refraction) $\therefore \dfrac{\sin\theta - \sin\phi}{\sin\theta + \sin\phi} = \dfrac{\mu - 1}{\mu + 1}$, or $\dfrac{\tan \frac{1}{2}(\theta - \phi)}{\tan \frac{1}{2}(\theta + \phi)} =$ constant. $\therefore$ since $\frac{1}{2}(\theta + \phi)$ may be taken to be θ, because $(\theta - \phi)$ is small; $\tan \frac{1}{2}(\theta - \phi)$ is proportional to $\tan\theta$, or the refraction is proportional to the tangent of the zenith distance.

the pole star above and below the pole (the sum of which equals twice the colatitude of the place). The sum of these four quantities must be 180° diminished by the sum of the four refractions, since the sum of the real zenith distances unaffected by refraction should be 180°; hence he obtained the sum of the four refractions, and then by theory apportioned the proper quantity of refraction to each zenith distance. In this manner he constructed his table of refractions. Thus let the zenith distances of the sun be Z and Z; and of the star ζ and ζ', then we have $(Z+K \tan Z) + (Z' + K \tan Z') + (\zeta + K \tan \zeta) + (\zeta' + K \tan \zeta') = 180°$; solving for K, the only unknown quantity in this equation, we have the coefficient of refraction.

51. The ancients made no allowance for refraction, although it was in some measure known to Ptolemy, who lived in the second century. He remarks a difference in the times of rising and setting of the stars in different states of the atmosphere. This however only shows that he was acquainted with a variation of refraction, and not with the quantity of refraction itself. Alhazen, a Saracen astronomer of Spain, in the ninth century, first observed the different effects of refraction on the height of the same star above and below the pole. Tycho Brahe, in the sixteenth century, first constructed a table of refractions. This was a very imperfect one.

52. As the atmosphere refracts light, it also *reflects* it, which is the cause of a considerable portion of the daylight we enjoy. After sun-set also the atmosphere *reflects* to us the light of the sun, and prevents us from being plunged into instant darkness, upon the first absence of the sun, just as in mountainous countries the sun's rays are reflected from the tops of the mountains into the valleys, before sun-rise and after sun-set, as it is seen from the valleys or adjacent plains. The inhabitants thus receive the light of the sun after he has disappeared below the horizon. Repeated observations show that we enjoy some twilight, till the sun has descended 18° below the horizon. From whence it has

been attempted to compute the height of the atmosphere, capable of reflecting rays of the sun sufficient to reach us; but there is much uncertainty in the matter. If the rays come to us after one reflection, they are reflected from a height of about 40 miles; if after two, or three, or four, the heights will be twelve, five, and three miles.

53. The duration of twilight depends upon the latitude of the place and declination of the sun. The sun's depression being 18° at the end of twilight, we are given the three sides of a spherical triangle, of which the angular points are the zenith, the pole, and the sun, to find an angle; viz., we are given the sun's zenith distance (90° + 18°), the polar distance, and the complement of latitude, to find the hour angle from noon, at which twilight begins. At and near the equator, the twilight is always short, the parallels of declination being nearly at right angles to the horizon, consequently the sun traverses the space between the horizon and a parallel to the horizon 18° below it, in the shortest line between these circles; in fact, this space is described in an hour and twelve minutes. At the poles the twilight lasts for several months; at the north pole from 25th January to 20th March. When the difference between the sun's polar distance and the latitude is less than 18°, the twilight lasts all night, since the sun in describing the small circle of declination never descends 18° below the horizon during the whole of the twenty-four hours.

54. Refraction is the cause of the oval figures which the sun and moon exhibit, when near the horizon. The upper limb is less refracted than the lower, by nearly five minutes, or $\frac{1}{6}$ of the whole diameter, while the diameter parallel to the horizon remains the same.[a] The

[a] Thus, when the sun's apparent diameter is 32′, if the true zenith distance of the upper limb be 84°28′, it will be raised through 8′46″, and the true zenith distance of the lower limb being then 85°, it will be raised through 9′46″, or 1′ more than the upper limb, so that in this case the vertical diameter will appear to be only 31′, while the horizontal remains 32′.

rays from objects in the horizon pass through a *greater* space of a *denser* atmosphere than those in the zenith, hence they must appear less bright. According to Bougier, who made many experiments on light, they are 1300 times fainter, whence it is not surprising that we can look upon the sun in the horizon without injuring the sight.

55. Another striking phenomcnon respecting the sun and moon in the horizon must not be entirely passed over, although rather belonging to the science of Optics, viz., their great apparent magnitudes. The cause of this, undoubtedly, is the wrong judgment we form of their distances then, compared with their distances when their altitudes are greater. In estimating their distances when in the horizon, we are led to judge them greater than when considerably elevated, partly from the number of intervening objects, and partly from the diminished brightness. The apparent diameters being nearly the same in both cases, we are apt to judge that object largest, the distance of which we conceive greatest. This explanation is a very old one, being given by Alhazen in the ninth century. Roger Bacon, Kepler, Des Cartes, and others also, were of the same opinion.

The following Table gives the Average Amount of Refraction for different Altitudes.

Altitude.	Refraction.	Altitude.	Refraction.
0	34′54″	15°	3′32″
1	24 25	20	2 37
2	18 9	30	1 40
3	14 15	40	1 9
4	11 39	45	58
5	9 46	50	48
6	8 23	70	21
7	7 20	90	0
8	6 30		
9	5 49		
10	5 16		

CHAPTER V.

MICROMETERS—DIAMETERS AND DISTANCES OF THE SUN, MOON, AND PLANETS—SPOTS ON THE SUN AND PLANETS—ROTATION OF THE SUN AND PLANETS—MAGNITUDES OF THE SUN, MOON, AND PLANETS.

56. HAVING attained to the knowledge of the magnitude and figure of the earth, we are enabled to extend our inquiries to the magnitudes and distances of the sun, moon, and planets. The present improved state of astronomical instruments furnishes means of making observations, by which we can obtain, with considerable precision, the magnitudes of the sun, moon, and planets, and ascertain the vastness of the distances of some of them, relatively to the diameter of the earth. We can ascertain the angle which two remote places on the surface of the earth subtend to a spectator at the sun, moon, or planets, and from thence deduce the angle the disc of the earth, when seen from any of these bodies, subtends. This angle can be obtained with the same accuracy as we can measure the apparent diameter of the disc of a planet. The method requires not the assistance of any theory of the arrangement of the celestial bodies, and therefore enables us to use the magnificent truths it furnishes in establishing the true planetary system. The fixed stars appear, as was observed, precisely in the same position with respect to each other, in whatever part of the earth we are; but the planets vary their position with respect to the neighbouring fixed stars, the angular distance of a planet from a neighbouring fixed star appearing greater in one place than in another. It is from the difference of these angular

distances that we obtain the angle which we should see the two places subtend, could we remove ourselves to the planet to make the observation.

57. Let us proceed to consider this method more particularly; but first it may be proper to make a few remarks respecting the method of measuring small angles on the concave surface, and on the precision with which they can be measured.

The diameters of the sun, moon, and planets—that is, the angles they subtend, can be measured with much accuracy, by measuring the diameters of their images, formed by the object glass of the telescope. The image is measured by means of two parallel wires placed in the focus of the object glass. One of these wires is capable of being moved parallel to itself, so that the wires may be readily opened to touch the opposite sides of the image of a planet's disc, and the interval of the wires furnishes at once the apparent diameter of the planet, the scale being previously settled by ascertaining the opening of the wires corresponding to a given angle. This is one of the simplest kinds of micrometers in its simplest state; there are others which it is unnecessary to mention here. The above is sufficient to give an idea of the method of measuring small angles. Small angles can be measured with much more accuracy than large angles. In measuring large angles the whole telescope is moveable. In micrometer measures, only the small apparatus of the wires is moveable, which can be executed with much greater nicety and exactness than the aggregate parts of a large instrument. The parts of the micrometer have much greater stability than the parts of an instrument for measuring large angles. Small angles may be measured, by good instruments, with certainty, to a fraction of 1″. The difference of declinations of two stars, having nearly the same declination, is also readily measured by moving the telescope, and turning the system of wires, so that one of the stars moves on the fixed wire, and then moving the other wire till the other star, when it enters the field of view of the telescope, moves along

it. This may be readily done, even if the stars differ considerably in right ascension, and consequently do not pass through the telescope at the same time, but are so near in declination, that they are both successively seen to pass through the telescope while it remains fixed. The angular distance between the wires gives the difference of the declination of the two stars.

If the stars differ considerably in right ascension, the quantity of refraction at each observation may be changed, on account of the variation of the barometer and thermometer, and must be allowed for; but when they are near together they are both equally affected by refraction, and therefore no allowance is necessary, which is a considerable advantage.

58. To find the angle two distinct places, in the same terrestrial meridian, subtend at a planet. Let H and S be the two places, P a planet in the celestial meridian of these places. HF′ and SF the directions in which the same fixed star, also in the meridian at the same time, is seen at the two places. The star made use of is supposed to be very nearly in the same parallel of declination as the planet—that is, not differing in declination more than a few minutes. Produce HP to meet SF in B : then because HF′ and SF are parallel (Art. 31) HBS = BHF′ : therefore HPS (= HBS + PSB) = F′HP + PSF = the sum of the apparent distances of the planet and star (the place of the planet being supposed to be between the parallels). These distances can be observed, as was said, with great accuracy, by means of a micrometer.[a]

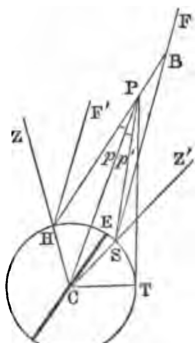

Fig. 9.

[a] The above is on the suppositions, 1st, that the star and planet are on the meridian together; 2nd, that the two places are on the same terrestrial meridian. If the *star and planet* are not on the meridian together, yet their difference of declinations being observed, it

59. In order to obtain the angle which the earth's disc subtends at the moon, or a planet P, let a tangent be drawn from P to the earth, and the angle CPT (Fig. 9) will be evidently half of that angle. Now, since it is very small, we may take it to be equal to its sine $\frac{CT}{CP}$. The angle CPT is called the horizontal parallax. Now, let $CH = r$; $CP = R$; $ZHP = z$; $Z'SP = z'$; then in the triangle HCP, hence $r : R :: \sin HPC : \sin (180° - z)$; therefore, sin HPC, or $\sin p$, or p (since it is small) $\frac{r}{R} \sin z$, and similarly from the triangle CPS, $p' = \frac{r}{R} \sin z' \therefore p + p'$ (or HPS) $= \frac{r}{R} (\sin z + \sin z')$; hence the horizontal parallax, or $\frac{r}{R}$, $= \frac{p + p'}{\sin z + \sin z'}$, where $p + p'$ is the angle found from observation, as explained in 58.

This is sometimes given in the following form:— $\frac{r}{R} (\sin z + \sin z') = z + z' - (l + l')$, where l and l' are the latitudes of the two observatories, since $p + p' = z + z' -$ HCS, or $= z + z' -$ (HCE + ECT), CE being the equator, or $\frac{r}{R} = \frac{z + z - (l + l)}{\sin z + \sin z'}$, which is the horizontal parallax.

60. The Cape of Good Hope is nearly in the same meridian with many places in Europe, having observatories for astronomical purposes, and therefore a comparison of the observations made there, with those made in Europe, furnishes us with a means of practising this method. By a comparison of the observations of De

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is the same as if there had been a star on the meridian with the planet. If the *two places* are not under the same meridian, an allowance must be made for the planet's motions in the interval between its passages over the two meridians, and if the difference in the longitudes of the two places be known, the zenith distances of the planet at one of the places of observation may be reduced to what it would have been had it been observed in the same latitude, but on the meridian of the other place.

La Caille, made at the Cape of Good Hope, with those made at Greenwich, Paris, Bologna, Stockholm, and Upsal, the angles the earth's disc subtends at Mars and at the Moon, have been obtained with very considerable precision. Comparisons of observations will also furnish the same for the sun and other planets. But it will be seen hereafter, that knowing the angle the earth's disc subtends at any one planet, we can readily find it for the sun or any other planet. If we seek the distance of Jupiter, or any planet outside his orbit, we are obliged to find the *annual* parallax of the planet, or *half* the angle which the earth's *orbit* subtends at the planet.

61. The method that has been described, yields only to one other method in point of accuracy; viz., that furnished by the transit of Venus over the sun's disc, which will be particularized hereafter. The above is fully sufficient for the purposes for which it is given here; which purposes are to enable us to compare the magnitudes of the sun and planets with that of the earth, and to show the vast distances of some of them relatively to the diameter of the earth.

The apparent diameter of the earth when nearest to and seen from

The Sun	is	17·86″	Juno } Vesta }	is	9″
Mercury	,,	28″			
Venus	,,	62″	Jupiter	,,	4″
Mars	,,	42″	Saturn	,,	2″
Ceres } Pallas }	,,	9″	Uranus	,,	1″
			The Moon	,,	2° 2′

A planet therefore appearing to us as small as the earth appears to the inhabitants of Saturn, Uranus, and Neptune, and would not have been observed except by the assistance of the telescope.

62. The Sun, Jupiter, Saturn, Uranus, and Neptune always appear with discs nearly circular.

The Moon, Mercury, Venus, and Mars, exhibit variable discs; they however are always portions of circles. Their diameters may be measured with micrometers, and are found to be, when greatest, as follow :—

The Sun	is	1956″	Saturn	is	18″
Mercury	,,	11″	Uranus	,,	4″
Venus	,,	57″	Neptune	,,	2″
Mars	,,	26″	The Moon	,,	1920″
Jupiter	,,	40″			

The asteroids, according to the most careful trials of Sir W. Herschel, appear to subtend only a small part of a second.

63. Hence we can compare the real diameters of these bodies with the diameter of the earth. For:—Diameter of planet: diameter of earth : : angle planet subtends at the earth: angle earth subtends from planet.

Whence calling the diameter of the earth unity, or 7,926 miles, the equatorial diameter of

		Diam. of ⊕		Miles.
The Sun	is	112	or	888,000 nearly.
Mercury	,,	0·37	,,	3000
Venus	,,	0·9	,,	7770
Mars	,,	0·56	,,	4500
Jupiter	,,	11.5	,,	92,000
Saturn	,,	9·4	,,	75,000
Uranus	,,	4·5	,,	36,000
Neptune	,,	4·4	,,	35,000
The Moon	,,	0·27	,,	2153

The largest of the asteroids is supposed not to exceed 300 miles in diameter.

64. The above method of obtaining the proportion of the diameter of a planet to that of the earth, admits of being repeated at pleasure, not being affected by the variableness of the planet's distance, and therefore a mean of many results being taken, great accuracy can be attained to.[a]

65. SUN'S ROTATION.—*Solar Spots.*—Having deduced

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[a] Knowing the angle the earth's disc subtends at the sun or a planet, we can ascertain the distance between the earth and that body, because the angle in seconds subtended by the earth : 206265″ (the seconds in arc equal radius) : : diameter of the earth in

the real magnitudes of the apparent circular discs, the next step is to show that the sun and planets are spherical bodies. With respect to the sun we are assisted by the consideration of its spots. By the help of telescopes we often observe, on the bright surface of the sun, dark spots of various and irregular forms, surrounded by a kind of borderless ark, called the penumbra. These appear to move on the surface from east to west, and after arriving at the western edge disappear, and after a time again reappear on the eastern edge. The times of appearance and disappearance are nearly equal, each being 13½ days nearly. The deduction to be made from these circumstances is, that the spots are on the surface of the sun, for they cannot be bodies revolving about him, for then they would not appear on his surface, and disappear during equal times. The time of appearance would be only that of passing over the sun's disc. The sun then must revolve on an axis carrying these spots with him, or these spots must move on his surface with such a motion as will account for the phenomena. The latter hypothesis is much more complicated than the former, for each spot separately must have such a motion given to it, as will solve the phenomena of its appearance and disappearance. The spots are not permanent, but are observed to increase and decrease, and at last cease to exist; nor are they stationary on the sun's disc, for the time which they take to cross it

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miles : distance of the planet from the earth in miles. But a small error in the angle subtended by the earth, will occasion a considerable error in the distances, and therefore this method of ascertaining the distance is not given, as affording much precision; but it serves sufficiently for showing the vast distances of the sun and planets from the earth, which is all that is necessary for our purpose here. If the angle subtended at the sun by the earth be 17·86″, the sun's distance from the earth is $\frac{206265''}{17.86''}$ (= 11746) times the diameter of the earth, or 93,098,796 miles.

In like manner taking 4″, 2″, and 1″ for the angles subtended by the earth's disc at Jupiter, Saturn, and Uranus, the distances of these planets from the earth will be 51566, 103132, 206265 diameters of the earth respectively. In this manner the mean distance of the moon from the earth is found to be about 60 *semidiameters* of the earth.

is not always the same. They appear to have a motion of their own relative to the sun's surface depending on the distance of the spot from the sun's equator; the time of the sun's rotation is, therefore, found to be different when derived from the motion of spots on the sun's equator, and at greater latitudes. In the former case it would be 25·2 days, and from the motion of a spot in latitude 45°, 27·8 days. The velocity too with which they appear to move across the sun is nearly that with which a point on the sun's surface would move if the sun revolved on its axis.

66. Concluding then that the sun revolves on an axis, we immediately deduce that it is a spherical body, for no revolving body but a sphere will always appear, at a distance, a circular disc. The motions of the spots show that the sun revolves on an axis inclined to the ecliptic at an angle of $82\frac{1}{2}°$, and that the time of revolution is $25^d\ 7^h$.

67. As the spots are occasionally seen by the naked eye, it is readily conceived they may be easily seen by the help of the most indifferent telescopes, having the eye, of course, protected by a dark glass from the brightness and heat of the rays of the sun. Accordingly, after the invention of that instrument, they soon became objects of much notice. The first discovery of them is contended for by Galileo, Scheiner, and Harriot. Harriot observed them in England in December, 1610, which was about the same time as when Galileo mentions that he had observed them. It was not long after they were first discovered that the inclination of the solar axis and time of revolution were ascertained. In modern times they are very regularly observed on account of the interest attached to them. Photographs of them are also taken daily, when practicable, at the Kew Observatory.

68. ROTATIONS OF THE PLANETS.—By the apparent motion of spots on the discs, as well as by other arguments to be mentioned hereafter, we know that the planets Mercury, Venus, Mars, Jupiter, and Saturn, are spherical bodies, each revolving on an axis.

Mercury revolves in	$24^h\ 5^m$	Jupiter revolves in	$9^h\ 5^m$
Venus " "	23 21	Saturn " "	10 29
Mars " "	24 37		

The rotation of Saturn was first ascertained from observation by Sir W. Herschel; that of Venus by M. Schroeter, a celebrated German astronomer.

69. No appearances have been discovered in the other planets sufficient to determine their rotation; but it is highly probable from analogy that they revolve on axes. But we have otherwise sufficient proof of their spherical form; for if they were merely circular discs or hemispheres, it is highly improbable that, their motions among the fixed stars being so irregular as seen from the earth, they would always keep the same face turned toward it; for the motions being observed to be sometimes direct, and sometimes retrograde, the planet, unless it be a spherical body, must, to preserve the same circular appearance, have contrary motions about the same axis.

70. The rotations of the sun and planets on their axes are all in the same direction—namely, that in which the planets revolve round the sun.

71. The sun and planets being spherical bodies, their magnitudes will be to that of the earth as the cubes of their diameters to the cube of the diameter of the earth; whence calling the magnitude of the earth unity, the magnitude of

The Sun is	1416428	Saturn is	770
Mercury "	0·059	Uranus "	96
Venus "	0·83	Neptune "	90
Mars "	0·183	The Moon "	$\frac{1}{64}$
Jupiter "	1412		

72. The ancients had such very inadequate notions of the magnitudes and distances of the sun and planets, that the earth was considered, by them, a body of as much importance as any other in the universe. Pythagoras, as may be collected from Pliny, considered the sun only three times more distant than the moon, and the moon thirteen times less distant than it is; hence, according to him, the sun was distant only by

seven diameters of the earth instead of 11746, and so the diameter of the sun would be only $\frac{1}{13}$ of the diameter of the earth. Aristarchus, in the third century before Christ, investigated the distance of the sun, and found it to be only 1200 diameters of the earth. Kepler, about two centuries ago, considered it nearly five times less distant than it is.

73. A spectator observing a planet not in his zenith, refers it to a place among the fixed stars, different from that to which a spectator, at the centre of the earth, would refer it. The place seen from the centre of the earth is called its true place: the arc of the great circle intercepted between these imaginary points is called the *diurnal parallax*.

74. Parallax.—The diurnal parallax is equal to the angle subtended at the planet by the place of the spectator and centre of the earth. For, to a spectator at H (Fig. 9, page 46), a fixed star in the direction HZ is in the zenith, and the distance of the planet P from this star is ZHP, but at the centre C the distance is ZCP, and the difference of these is the angle HPC (p). We have seen before that $p = \frac{r}{R} \sin Z$, and it is therefore greatest when Z is a right angle—that is, when the planet is on the horizon. The parallax of a planet, when in the horizon, is called the *horizontal parallax*, and is equal to the angle the semi-diameter of the earth subtends at the planet.

75. The diurnal parallax *depresses* an object, or diminishes its altitude; a planet, at rising, appears to the eastward of its true place, and at setting, to the westward, whence the term diurnal parallax. The sun, moon, and planets are affected by diurnal parallax. The stars are not, as they are at such an immense distance that lines drawn from the centre and from a point on the surface of the earth to the same fixed star are parallel—that is, a star is seen in the same position as it would be seen from the centre of the earth. The greater the distance of a planet the less is its parallax.

CHAPTER VI.

THE ROTATION OF THE EARTH—PENDULUM EXPERIMENT—MOTION OF THE EARTH ABOUT THE SUN—GREAT DISTANCES OF THE FIXED STARS—PRECESSION OF THE EQUINOXES.

76. HAVING acquired a knowledge of the vast distances of the sun and planets, and of their magnitudes, we are led to consider whether the diurnal motion we observe in these bodies be a real or only an apparent motion. Real and apparent motions are not at first readily distinguished from each other. The motions of a person in a ship, carriage, &c., daily afford instances that vision alone is not sufficient to distinguish between true and apparent motion. Either experience or judgment is necessary to distinguish between them.

DIURNAL MOTION.—That the heavenly bodies really move, and, by doing so, cause the apparent *diurnal* motion, we can have no experience, nor can we readily perceive the motion of our earth, as we in that respect are in the same circumstances as a person in the cabin of a ship in motion. We could not easily understand whether the whole motion was in the ship, or in a bird (the only visible external object), flying at a distance. But examining the reasons for each, we distinguish which motion is most probable, that of the earth round its axis, or of all the celestial bodies in the space of $23^h\ 56^m$. Either the celestial bodies revolve in the space of $23^h\ 56^m$ in great or small parallel circles, according to their apparent distance from the celestial poles, or the cause of that apparent diurnal motion is a real motion of the earth about an axis in a direction from west to east. That the latter supposition will

explain the diurnal phenomena is so evident, that it is hardly necessary to dwell upon it. By the rotation of the earth about an axis, the horizon of each spectator, which is the tangent plane to the earth's surface at that place, has a motion, and its intersection with the heavens, which is the celestial horizon, will revolve in the celestial sphere, so that the parts of the sphere will be successively uncovered by it, and become visible, just as they would do by a motion of the imaginary sphere itself, with its circles round the earth at rest, carrying the sun, moon, and stars situate in it round the spectator once in $23^h\ 56^m$.

77. The only argument against this motion is, that the spectator appears at rest, and the celestial bodies appear to move. But as experience every day points out to us motions only apparent, nothing can be concluded from the apparent rest of the spectator. The arguments from analogy in favour of the rotation of the earth are very strong. The Sun, Venus, Mars, Jupiter, and Saturn, all spherical bodies like the earth (of which, three are vastly greater than the earth), are known to revolve about their axes; consequently, we may conclude that it is most probable that the earth moves in the same way.

78. Also against the diurnal motions of the celestial bodies about the earth, are the vast distances and magnitudes of the sun and planets. The immense motions to be given to each of these bodies at different and variable distances from the earth, and apparently unconnected with each other and with the earth, to produce their apparent diurnal motions, would require a very complicated celestial mechanism. To suppose the sun above a million times larger than the earth, to revolve about the earth in 24 hours, instead of the earth revolving about an axis in that time, is contrary to that rule of philosophy by which effects are deduced from the simplest causes.

79. Also we know that when a body moves in the circumference of a circle, there is requisite a force tending to the centre to keep it continually in that

circle. It is proved by writers on mechanics that the centrifugal force, generated by uniform motion in a circle, is equal to $\frac{4\pi^2 R}{T^2}$ multiplied by the mass of the body, where R is the radius of the circle, and T the time of revolution. This must be counteracted in the case of unconstrained motion by a force equal and opposite tending towards the centre of the circle. Now, we can assign no force acting upon the sun and planets, to make them describe the diurnal circles. No bodies are situate in the different centres of those circles, by the continual attraction of which they might be continually impelled from the tangent to the circumference.

80. EXPERIMENTAL PROOFS OF EARTH'S ROTATION.—Although the arguments for the rotation of the earth are so satisfactory, that no doubt whatever can remain; yet it is interesting to consider whether the matter cannot be subjected to a direct experiment. It will readily appear that a body let fall from a considerable height will, if the earth revolves from west to east, fall to the eastward of the vertical line. Let C (Fig. 10) be the centre of the earth, T the place from which the body is let fall, TB the vertical line in direction of the centre. When the body reaches the earth, let *tb* be the position of the vertical line, in consequence of the earth's motion. Take B*f* = T*t* and *f* will be the place of the body; because the body, leaving the top of the vertical with a motion equal to the motion of the top, is, at the end of its fall, as far from the first position of the vertical as the top of the vertical itself is from its first position. But B*b* is less than T*t*, and therefore than B*f*, in the proportion of CB to CT; consequently, *f* is to the eastward of *b*. This is on the supposition that the place is at the equator, and it may suffice for an illustration. On account of the small height BT, at

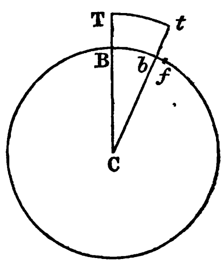

Fig. 10.

which we can make the experiment, *bf* must be very small, and the utmost nicety is required : in this age, however, of accurate experiment, it has been attempted, and, it is said, with success. It has been tried at Bologna from the height of 257 English feet; also at Viviers and Hamburgh. At Hamburgh the height was 250 feet, and the deviation found to be 0·35 inches to the east. Computation, not taking into account the air's resistance, gives 0·34 inches to the east.

81. PENDULUM EXPERIMENT.—In the year 1851, M. Fourcault suggested an experiment, by which certain effects of the earth's rotation might be rendered visible. If a heavy ball be suspended from a point by a string, without any twist, and a motion be given to it by drawing it aside from the vertical, and allowing it to drop gently towards the position of equilibrium, it will oscillate, as a pendulum, and the time of each oscillation will be proportional to the square root of the length of the string; consequently, the longer the distance of the ball from the point of suspension, the slower, and consequently more easily measurable, will be its motion. The plane in which it vibrates will remain always fixed in space, unless some external force different from gravity acts upon the ball, and tends to make it deviate from that plane. Now, it may be shown by an easy experiment that, if a hoop be constructed, capable of revolving round a vertical diameter, and a string, by which a ball is suspended, be attached to the highest point of that diameter, and a pendulous motion be given to the ball as above, a rapid motion of rotation communicated to the hoop round an axis passing through the point of suspension of the pendulum, will in no way affect the plane in which the pendulum itself moves. This is exactly what would take place if an observer could reach the north pole of the earth, and suspend such a pendulum from a point on the earth's axis of rotation; the plane of vibration of the pendulum would remain fixed in space, unaffected by the earth's motion; consequently, the earth would revolve under that plane from west to east; but as the observer is unconscious of his own motion

along with the earth, the plane of vibration of the pendulum would appear to him to move round in the opposite direction from east to west, and to move through all the points of the compass in $23^h\ 56^m$.

Next, let us suppose such a pendulum to be set in motion at the earth's equator, the parts of the earth there have no motion of rotation round the equatorial radius drawn to the earth's centre; the ball during the whole of its oscillation partakes of the motion of all the points of the equator round the polar axis, and in whatever direction it is set in motion there is nothing to disturb the relative position of the plane of oscillation to the earth's surface; consequently, it appears unaltered during the whole time of the earth's rotation.

Let the point of the earth's surface, at which the pendulum is set in motion, be between the equator and the pole. In order to determine the effect of the earth's rotation upon such a pendulum, we must bear in mind that writers upon mechanics have shown that, just as the effect of a force may be resolved into the simultaneous effects of two component forces, acting at right angles to each other in the directions of the sides of a rectangle of which the diagonal is the direction of the original force, and as the magnitude of each of which is found by multiplying that force by the cosine of the angle between the diagonal and the side of the rectangle; so, in like manner, a motion of rotation, which is given to a body round an axis, is equivalent to the joint effects of two component rotations round two axes at right angles to each other, and in the same plane with the original axis; and if the angular velocity round this axis be ω, it may be shown that the component angular velocities will be $\omega \cos \alpha$ and $\omega \cos \beta$, where α and β $(= 90^\circ - \alpha)$ are the angles which the component axes make with the original one. In this way the rotation of the earth round the polar axis may be resolved into two component rotations; one round an axis, passing through a place whose latitude is λ, and another round an axis at right angles to this latter; and if the angular velocity of the earth in revolving round the polar axis be $\frac{2\pi}{23^h\ 56^m}$, the component

angular velocity round the axis passing through the given place will be that angular velocity multiplied by the cosine of the angular distance of the place from the north pole, or $\frac{2\pi}{23^h\,56^m} \times \sin\lambda$. Now, these resolved rotations take place simultaneously, and with regard to one of them, the given place is, as it were, at the pole, and with respect to the other, at 90° from the axis, and therefore at the corresponding equator. So that if a pendulum be set in motion at the place its plane of vibration ought to be affected by the first, but not by the second of these rotations. The time of rotation is equal to the 2π divided by the angular velocity; consequently, the time of rotation round the axes through the place is $\frac{23^h\,56^m}{\sin\lambda}$, or $23^h\,56^m \times \operatorname{cosec}\lambda$. The plane of vibration of the pendulum ought to appear to move round relatively to surrounding objects in this time, and in smaller times through proportionate angles. This is found by experiment to be strictly the case as long as the motion of the pendulum continues. This seeming change in the plane of the vibration of the pendulum can be explained only by the earth's rotation round its axis.

We may conclude, then, that the diurnal motions of the celestial bodies are only apparent, and that these appearances are produced by the motion of the earth about an axis parallel to the apparent celestial axis; although every appearance may be explained by supposing the eye in the centre of a revolving sphere, in the concave surface of which the heavenly bodies are situate.

82. The rotation of the earth has been established, beyond all controversy, since the time of Galileo; but the notion is a very old one. It is expressly mentioned by Cicero as the opinion of Hicetas, who lived about 400 years before the commencement of our era. The words of Cicero are:—"Hicetas Syracusius, ut ait Theophrastus, cœlum, solem, lunam, stellas, supera denique omnia stare censet; neque præter terram rem ullam in mundo moveri: quæ cum circum axem se summâ celeritate convertat et torqueat, eadem effici

omnia, quasi stante terrâ cœlum moveretur." *Acad. Quæst. Lib.* 2. Copernicus states that these words led him first to think of the earth's motion.

83. Annual Motion.—The apparent annual motion of the sun is explained, by supposing that either the sun moves round the earth, or the earth round the sun, in a path or orbit nearly circular. For the sun, as has been stated, appears in the course of a year to describe, on the concave surface of the heavens, a great circle called the ecliptic. Observation shows that its apparent diameter does not vary much, its greatest being = 32′34″, and least 31′29″; consequently the variation of distance, compared with the whole distance, is but small. Observations likewise show that its apparent motion in the ecliptic or change of longitude is not equable, yet its difference from equable motion is not great. The motion for any given interval of time, if it moved equably, is found by dividing its whole motion in a year by the number of given intervals in a year. Thus it moves 360° in about 365 days, therefore in an hour the motion is 2′28″ nearly. This is called *the mean motion* in an hour. Its greatest hourly motion is 2′33″, and its least 2′23″. Whence in a year the sun moves in an orbit nearly circular, and with a motion nearly equable, about the earth, or the earth moves in an orbit nearly circular, with a motion nearly equable, about the sun. That the latter motion takes place is established by a variety of reasons.

84. It will be proved that the planets move about the sun in orbits nearly circular, in different periodic times, and at different distances. Also, that all the planets receive their light from the sun—a body vastly greater than them all in magnitude, some of which are of much greater magnitude than the earth. Again, it will hereafter appear that the squares of the periodic times of the planets are proportional to the cubes of their distances from the sun. Now, considering the earth as a planet revolving round the sun, its distance and periodic time obey the law of the rest of the planets: which circumstance affording such harmony between

the motions of all those bodies, receiving from the sun their light, and apparently their heat, the source of animal and vegetable life, must at once persuade us to acknowledge the annual motion of the earth, rather than of the sun: although all the principal phenomena of the planetary motions may be explained, by supposing them, as Ptolemy did formerly, to revolve in orbits nearly circular round the sun, while the sun and planets are together carried with an annual motion round the earth.

85. But the most satisfactory proof is one that we cannot introduce with its full effect here, it requiring some preliminary principles of physical astronomy. This proof is from the knowledge of that universal attendant of matter, the principle of attraction or gravity. The sun, earth, and planets mutually attract each other, in proportion to their quantities of matter or their masses. It follows, from the laws of motion, that they must come together, or each of them revolve in an orbit round a fixed point, the common centre of gravity of all the bodies. Now we shall see hereafter that the *mass* of the sun, as well as its *magnitude*, is vastly greater than of all the planets together, so much greater, that the common centre of gravity lies within the body of the sun; and the sun, in fact, will move about this point, but in a path so small, compared with the orbits of the planets, that it may be said to be at rest, and the planets said to revolve about the sun, they revolving about a point so near his centre.

Another argument, derived from the aberration of the fixed stars, will be mentioned hereafter.

86. But it is necessary to show how this annual motion will explain the changes of the seasons, or rather how the annual motion of the earth will explain the apparent motion of the sun in a great circle inclined to the equator; for from this, as we have seen, are explained the changes of seasons.

The annual motion of the earth in an orbit, the plane of which passes through the sun, is independent of its motion round the axis. That a globe may have two

motions independent of each other, one a progressive motion equally affecting each particle, and the other a rotatory motion about an axis, is easily shown from mechanical principles. As the progressive motion affects each particle equally, it cannot affect the rotation of the globe about its axis, and therefore this axis will, while the globe has a progressive motion, remain parallel to itself. Supposing, then, the earth to have two such motions, it is clear that the axis of rotation cannot be perpendicular to the plane of the progressive motion, for otherwise the sun, which is always seen in this plane, would at all times appear in a plane at right angles to that axis—that is, in the celestial equator. But if the polar axis be inclined to the plane of the earth's orbit constantly at an angle of 66° 32′, a spectator any where on the earth will see the sun, in the course of a year, apparently describe on the surface of the celestial sphere a great circle, inclined to the equator at an angle of 23° 28′. For the plane of the earth's orbit, or that in which it moves round the sun, constantly making the same angle with the terrestrial equator (since the angle between two great circles is that between their planes, and this is the complement of the angle between one of them and a line perpendicular to the other, in this case between the plane of the ecliptic and the earth's axis), it will intersect the surface of the heavens in a great circle, inclined to the equator at an angle of 23° 28′, and therefore an eye at the centre of the earth will refer the place of the sun always seen in the plane of the orbit, to a great circle in the celestial sphere, which circle it will evidently appear to describe in the course of a year to an eye at the centre. But it was before shown, that, from the vast distance of the sun compared with the diameter of the earth, all spectators refer the sun nearly to the same place on the concave surface; whence we conclude that, by the motion of the earth about the sun in an orbit, to which the equator is inclined at a constant angle of 23° 28′, the sun, seen from any part of the earth, will appear to describe, in the space of a year, the great circle called the ecliptic.

87. EXPLANATION OF THE CHANGE OF THE SEASONS.—The effects also of this inclination and parallelism of the axis, will readily appear, by considering that a hemisphere (or rather somewhat more) of the earth, the base of which is perpendicular to the line joining the centres of the sun and earth, is illuminated by the sun. The positions of the poles and parallels of latitude with respect to this hemisphere, will easily show the variation of the length of the days and of the seasons.

Let (Fig. 11) represent an oblique view of the path

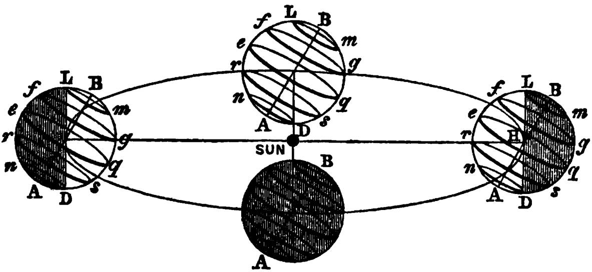

Fig. 11.

or orbit of the earth about the sun; let also AB represent the axis of the earth in four positions, B being the north and A the south pole. Conceive this axis in a plane at right angles to the orbit, and that this plane always continues parallel to itself, while the centre of the earth moves about the sun, the axis will then, it is evident, also move parallel to itself. The reader will mark the four positions of the earth's centre by H, H_1, H_2, H_3. Let AHB be the position of the axis when this plane passes through the sun, S, and the angle $rHB = 90° + 23° 28'$. When the centre H has moved a right angle about the sun to H_1, this imaginary plane being parallel to its former position, SH_1 must be at right angles to it—that is, to every line in it, therefore the angle between the earth's axis and the radius of its orbit is a right angle. When the centre comes to H_2 opposite to H, the plane again passes through the sun, and this angle $= 90° - 23° 28'$, and is then least. When it

comes to H_3 opposite to H_1, it is again a right angle. H (at the right hand of Fig, 11) will represent the place of the earth at the winter solstice, H_1, at the top, at the vernal equinox, H_2, at the left, at the summer solstice, and H_3, at the bottom, at the autumnal equinox. For the first position (at H) will represent the earth with its enlightened and dark hemispheres, seen at right angles to the plane of the meridian passing through the sun. The angle SHB is greater than in any other position, and the north pole B will be in the dark hemisphere farthest removed from the circle of light and darkness. The parallel of lat. *Lm* is the arctic circle, and will just touch the circle[a] of light and darkness. All places on the north side of the equator will have a greater portion of their parallels of latitude in the dark than in the enlightened hemisphere, and therefore the days will be shorter than the nights. The equator is equally divided, and the parallels on the southern side have a greater portion in the enlightened than in the dark hemisphere. *rs* will be the parallel to which the sun is vertical, and will represent the southern tropical circle, because $rHe = LHB = 23° 28'$.

H_1 (at the top of Fig 11) will be the place of the earth at the vernal equinox; for it will represent the earth at H_1 with its enlightened and dark hemispheres, viewed at right angles to the plane of the meridian passing through the sun. The circle of light and darkness will pass through the poles, and equally divide the parallels of latitude; therefore all places will have equal day and night, and the sun will be vertical to the equator.

H_2 (at the left hand side of Fig. 11) will be the place of the earth at the summer solstice; for it will represent the earth with its enlightened and dark hemispheres viewed as before at H, and the same may be remarked with respect to the northern and southern hemispheres, as was observed with respect to the southern and northern when the earth was in the first position at H.

[a] The circle called the circle of light and darkness, is the circle which is a boundary between the dark and enlightened hemispheres.

H_3 (opposite to H_2) will represent the earth when at the autumnal equinox.

88. An objection to the motion of the earth must be considered here, which, at first sight, may appear to have some weight. No change is observed in the relative position of the fixed stars, in consequence of that motion. The angular distances of the fixed stars, observed at different seasons of the year, always remain the same, even when observed with the most exquisite instruments. But, supposing the motion of the earth in an orbit, nearly circular, round the sun, the observer in one situation is nearer some stars by 23,500 diameters of the earth (*vide* note, page 50), than in another, and consequently the angular distances of those stars ought to appear greater.[a] Or just as a planet

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[a] Let TE (Fig. 12) represent the orbit of the earth, T and E the places of the earth at the solstices, when the axes Pp, P′ p′ of the earth are in a plane which passes through the sun, and is perpendicular to the plane of the orbit. This plane evidently passes through the solstices, and cuts the celestial sphere in the *solstitial colure.* Let F be a fixed star in this perpendicular plane. When the earth is at T, the observed distance of the star from the celestial pole is FTP; when at E it is FEP. Produce pP to meet FE in R: then the angle F = TRE − FTR = FEP − FTP. But these angles are constantly the same, not having any perceptible difference, and therefore the angle subtended by the diameter of the earth's orbit, at a star situate in the solstitial colure, is imperceptible. Dr. Bradley took much pains to ascertain the angle F in the case of γ Draconis—a star of the second magnitude, situate nearly in the plane above-mentioned, or in the solstitial colure, about 15° from the pole of the ecliptic. This star, passing the meridian near his zenith, admitted of being observed by a zenith sector, an instrument particularly adapted for observing with great precision near the zenith, where also no error can occur from the uncertainty of refraction. He found the angle F imperceptible by his observations. Dr. Brinkley's observations, and those of Mr. Pond—a former Astronomer Royal—agree also *as to this star,*

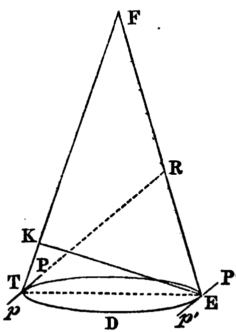

Fig. 12.

is referred to different places in the heavens, as seen from places near the north pole and the south pole of the earth; so also a fixed star ought to be seen in different places, as observed from places in the earth's orbit diametrically opposite—that is, just as there is the *horizontal* parallax of a planet—namely, the angle which the earth's radius subtends at a planet, so there ought to be the *annual* parallax of a fixed star—namely, the angle which the radius of the earth's *orbit* subtends at the star.

88. The first astronomer, who determined with certainty the annual parallax of a star, was Bessel, of Kœnigsburg, who found the parallax of 61 Cygni to be 0″.35. The method which he adopted was the following:—In order to determine the parallax of a star, which is supposed to have one, he compared its position with that of a star which is at such an immeasurable distance, that its position is known to be not altered when seen from opposite points of the earth's orbit; these two stars being very near each other, they are affected in the same manner by refraction, precession, nutation, and aberration. The distance between these stars is accurately measured during an entire year, and the greatest difference between their distances, as seen from opposite parts of the earth's orbit, at a distance of half a year, is ascertained; this difference is altogether due to the parallax of one star, as the other is so extremely distant, that lines drawn from opposite positions of the earth's orbit to it may be considered to be parallel.

The parallax of other stars has since been ascertained. The largest parallax is that of *a* Centauri, in

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in showing that the angle F is imperceptible. As an extreme case let us suppose the angle F to be even 2″, draw the perpendicular EK, then 2″ : 206265″ (the seconds in an arc = radius) : : sin 2″ : rad : : EK : FE. But EK : TE : : sin ETK : radius. For γ Draconis the angle ETK = 75° nearly; hence EK = 0.97.TE, and therefore $FE = \frac{206265}{2} \times 0.97.TE = 100000.TE$ nearly. If the earth therefore move about the sun, the distance of γ Draconis must be at least 200000 times greater than the distance of the sun from the earth.

the southern hemisphere, which is consequently supposed to be the star nearest to the earth. In the case of this star, it amounts to nearly 1″, or the angle which the diameter of the earth's orbit subtends at that star is 2″, and its distance would be 224,201 times the earth's distance from the sun. Light would take 3½ years to travel to us from this star. 61 Cygni is distant from us 458,366 times the sun's distance, and Sirius (the parallax of which is 0″.23) 896,803 times the same distance. In the latter case light would take more than 14 years to reach the earth from the star.

Yet there is nothing contrary to our reason or experience in admitting this almost inconceivable distance. Why should we limit the bounds of the universe by the limits of our senses? We see enough in every department of nature to deter us from rejecting any hypothesis, merely because it extends our ideas of the creation and the Creator.

The best telescopes do not magnify the fixed stars, so as to submit their diameters to measurement; but it is well ascertained that the apparent diameter of the brightest of them is less than 1″. Now, being self-shining bodies, and not subject, except in a few instances, to any apparent alteration, we may conclude them to be bodies of the nature of our sun. But that the diameter of the sun may appear less than a second, it must be removed 1900 times farther from us than at present; which is an argument in favour of the vast distance of the fixed stars. It must, however, be confessed that this argument from analogy is much too weak to be in any degree decisive, and our positive knowledge of the immense distance of the fixed stars must depend upon the certainty of our knowledge of the earth's motion, of which we have such evidence as must be considered conclusive.

89. Precession of the Equinoxes.—Although the place of the celestial pole among the fixed stars has been considered as not changed by the annual motion of the earth, yet in a long period of time it is observed to be changed, and also the situation of the celestial

equator; while the ecliptic retains the same situation among the fixed stars. Observation shows that this change of situation of the pole and equator is nearly regular. The pole of the celestial equator appears to move with a slow and nearly uniform motion, in a lesser circle, round the pole of the ecliptic; while the intersections of the equator and ecliptic move backward *on the ecliptic*, with a motion nearly uniform. This motion is at the rate of about 1° in 72 years, or more accurately 50″.2 in a year; consequently, since this intersection moves back to meet the sun in his annual path, the sun returns again to the same equinoctial point before he has completed his revolution in the ecliptic; so that the equinoxes *precede* continually the complete apparent revolution of the sun in the ecliptic; and hence the term precession of the equinoxes. In consequence of this apparent motion all the fixed stars increase their longitudes by 50″.2 in a year, and also change their right ascensions and declinations. Their latitudes remain the same, since the ecliptic remains very nearly in the same position with regard to them. The period of the revolution of the celestial equinoctial pole about the pole of the ecliptic is nearly 26,000 years.

The north celestial pole, therefore, will be about 13,000 years, hence nearly 49° from the polar star; and about 10,000 years hence, the bright star α Lyræ will be within 5° of the north pole. This star, therefore, which now, in these latitudes, passes the meridian within a few degrees of the zenith, and twelve hours after is near the horizon, will then remain nearly stationary with respect to the horizon. All which will readily appear, from considering the celestial concave surface as represented by a common celestial globe.

90. This motion of the celestial pole originates from a real motion in the earth, whereby its axis, preserving very nearly the same inclination to its orbit, has a slow retrograde conical motion. The cause of this motion is shown, by physical astronomy, to arise from the attraction of the sun and moon on the excess of matter

at the equatorial parts of the earth, arising from its spheroidal shape, by which it bulges out at the equator. This attraction is greater on the part of the equatorial ring of the earth nearest to the sun (or moon) than on the portion farthest from it, and consequently it produces a tendency to a rotation of the earth round an axis in the plane of the equator; the composition of this rotation with that round the axis passing through the poles, produces a resulting rotation round an axis between them, or alters the *direction* of the axis round which the earth is moving. By physical astronomy we are also enabled to account for a small change in the plane of the ecliptic. Observations, separated by a long interval, point out that the obliquity of the ecliptic is diminishing at nearly the rate of half a second in a year. Physical astronomy shows that this arises from a change in the plane of the earth's orbit, occasioned by the action of the planets: that this change of obliquity will never exceed a certain small limit; and that by this action of the planets the ecliptic is at present progressive on the equator $11''.5$ in a century.[a]

NUTATION.—The precession of the equinoxes is not en-entirely uniform, for a small inequality in the precession, and change in the obliquity of the equator to the ecliptic, depending on the position of the moon's *nodes* (the intersections of its path and the ecliptic) were discovered by Dr. Bradley, and are confirmed by physical astronomy. The poles of the equator describe round their mean places a small ellipse, not differing much from a circle, about $18''$ in diameter, in $18\frac{2}{3}$ years.[b]

91. The complicated motion of the pole of the

[a] Hence the annual precession arising from the action of the sun and moon on the spherical figure of the earth—that is, the luni-solar precession is $50''.35$ annually. The precession produced by the planets is $0''11.$ in the opposite direction; hence the general precession is $50''.24$, or rigorously $50''.35 - 0''.11 \cos \omega$.

[b] The inequality mentioned above is strictly called the lunar nutation. There is also a small solar inequality of precession, depending on the place of the sun in the ecliptic, and called the solar nutation. This can increase or diminish the longitudes $1''.3$.

equator round the pole of the ecliptic may be graphically represented as follows:—(Fig. 13) Let Π be the pole of the ecliptic, and M*ao*N a small circle, the distance of every point of which from Π is 23° 28′, then a curved wavy line *adobc*P*e*, such that Πb exceeds $\Pi\, d$ by 18″, will represent the motion of the pole of

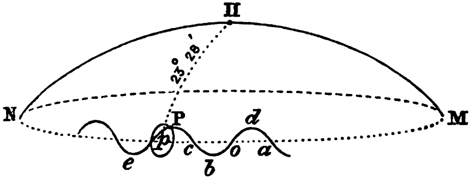

Fig. 13.

the equator P. Its motion will be the same as if it were moving in an ellipse, whose major axis, 18″·5, is always directed towards Π, and its minor axis 13″·7 lies in the direction *p c*, and completing this motion in 18$\frac{2}{3}$ years; while the centre of this ellipse *p* is at the same time steadily moving along the small circle *a o c p*, at the rate of 50″·2 in a year; the latter moti n is called the *precession*. The deviation from this u o fom motion, and the varying distance of the pole of the equator from the pole of the ecliptic, caused by the concurrent motion in the moveable ellipse, is called *nutation*. The distance *bc*P*e* is traversed in 18$\frac{2}{3}$ years. The precession of the equinoxes was first discovered by Hipparchus. As the quantity of it is so perceptible in a hundred years, a comparison of the position of the circles of the sphere, as recorded in the earliest era of astronomy, and of their position now, has been used to assist chronology, by determining the number of years which must have elapsed from the time when the equinoctial colure (or the great circle passing through *the poles and equinox*) passed through the middle of Aries, taking into account its motion of 1° in 72 years. Newton supposes that this defines the time of the Argonautic Expedition for which the celestial sphere was first constructed by Musæus, the argonaut.

CHAPTER VII.

ON THE MOTIONS OF THE PRIMARY PLANETS—THE SOLAR OR COPERNICAN SYSTEM.

92. HAVING stated some of the principal arguments for the motion of the earth in an orbit nearly circular about the sun, let us now consider the planets in general. Astronomy has added much indeed to our knowledge of the creation, by enabling us to ascertain that the planets are vast bodies, revolving round the sun in orbits nearly circular, some at greater, and others at less distances than the earth; that some of these bodies are smaller, and others much larger than the earth; and that, according to a high degree of probability, they are bodies of the same nature as that on which we live.

93. The principal planets are always observed to be nearly in an ecliptic, the annual path of the sun on the concave surface; and for the present let us consider them as seen in the ecliptic.

The most striking circumstance in the planetary motions is the apparent irregularity of those motions, the planets one while appearing to move in the same direction among the fixed stars as the sun and moon, at another in opposite directions, and sometimes appearing nearly stationary. These irregularities are only apparent, and arise from a combination of the motion of the earth, and motion of the planet; the observer, not being conscious of his own motion, attributing the whole motion to the planet.

94. The planets really move, according to the order of the signs of the zodiac, in orbits nearly circular, and with motions nearly uniform, round the sun in the centre, at different distances, and in different

periodic times. The periodic time is greater or less, according as the distance is greater or less. It may be shown that this motion of the planets round the sun agrees completely with their observed positions. For upon the hypothesis that the planets thus move, we can ascertain, by observation, their distances from the sun, and thence compute, for *any time*, the place of the planet with respect to the sun, which is *always* found to agree nearly with observation. This is effected in the following manner:

95. First, for those planets which are limited in their elongation from the sun—namely, Mercury and Venus. The *elongation* of a planet from the sun is the angle subtended at the earth by the sun and planet. These planets are nearer the sun than the earth is, and therefore called *inferior* planets. The greatest elongation of the inferior planet Mercury from the sun is about 28°, and the average value of the greatest elongation of Venus is about 46° 20′. When such a planet is at the greatest elongation, the line drawn from the earth to the planet is at right angles to the radius of its orbit; consequently, the distance of Venus from the sun : distance of the earth from the sun as sin 46° 21′ : 1.

The interval of time between two successive inferior conjunctions with the sun can be observed. A planet is said to be in *inferior conjunction*, when it comes between the sun and the earth. In *superior conjunction*, when the sun is between the earth and the planet. In inferior conjunction, the planet being nearest to the earth, appears largest, and may be observed with a good telescope, even a very short time before the conjunction. For our purpose here it is not necessary that the time of conjunction should be observed with great accuracy. Let T represent the time between two successive inferior conjunctions. Then, to a spectator in the sun, in the time T, the inferior planet (moving with a greater angular velocity) will appear to have gained four right angles, or 360°, on the earth; and the planet and earth being supposed to move with uniform

velocities about the sun, the angle gained (the angle at the sun between the earth and planet, reckoning according to the order of the signs of the zodiac) will increase uniformly.

96. Let TEL represent the orbit of the earth, DNPGO that of an inferior planet, each being supposed circular, S the sun in the centre, and P the place of the planet when the earth is at E. Then in the triangle SEP (Fig. 14) we obtain the angle SEP the elongation by observation,[a] and the angle PSE by computation, for it is the angle the planet has gained on the earth since the preceding inferior conjunction. Therefore, this angle PSE : 360° : : time from inferior conjunction : T. The two angles SEP and PSE being known, the angle SPE is known, and hence the ratio of SP to SE ; for sin SPE : sin SEP : : SE : SP. Having thus obtained the *ratio* of the distance of the planet from the sun, to that of the earth from the sun, we can, *at any time*, by help of the time T, and the time of the preceding inferior conjunction, compute the angular distance of the planet from the earth, as seen from the sun, and thence,

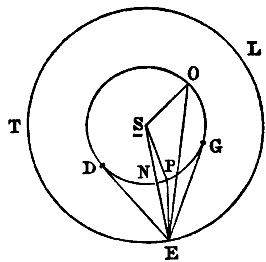

Fig. 14.

[a] The ancients observed the places of the fixed stars and planets with respect to the sun, by the assistance of the moon or planet Venus. In the day time they very frequently could observe the situation of the moon with respect to the sun. Venus also being occasionally visible to the naked eye in the day time, they used that star for the same purpose. Now we can, owing to the convenience of our instruments, without the intervention of a third object, obtain the angular distance of a planet from the sun, by observing the declinations of each, and the difference of their right ascensions. By which we have, in the triangle formed by the distances of each from the pole of the equator, and from each other, two sides, the two polar distances, and the included angle, the difference of right ascensions, to find the third side, the angular distance of the planet from the sun.

by help of the planet's and earth's distances from the sun, compute what the planet's elongation from the sun ought to be at that time. Thus the planet being at O, and the earth at E, we can compute the angle ESO; and having the ratio of the sides SE and SO, we can, by trigonometry, compute the angle SEO the elongation of the planet from the sun; since, if the vertical angle of a triangle and the *ratio* of the sides be known, the base angles can be calculated, inasmuch as we know the *sum* of the base angles, and the ratio of their sines. This *computed* elongation being compared with the *observed* angle, we always find them nearly agreeing, and thereby it is shown that the motions of the inferior planets, Mercury and Venus, are accounted for by those planets being supposed to move in orbits nearly circular about the sun in the centre. As the computed place always agrees with the observed place, it necessarily follows that the retrograde, stationary appearances, and direct motions of these planets, are explained by assigning these circular motions to them.

97. It is easy *to demonstrate the retrograde and stationary appearances.*

To do this more clearly, it will be necessary to consider the effect of the motion of the spectator arising from the motion of the earth, in changing the apparent place of a distant body. The spectator, not being conscious of his own motion, attributes the motion to the body, and conceives himself at rest. Let S be the sun (Fig. 15), ET the space described by the earth in a small portion of time, which may therefore be considered as rectilinear. The motion is from E toward T. Let V be a planet, supposed at rest anywhere on the same side of the line of

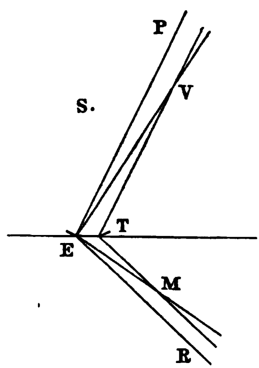

Fig. 15.

the direction of the earth's motion as the sun. Draw EP parallel to TV, then while the earth moves through ET, the planet supposed at rest will appear to a spectator, unconscious of his own motion, to have moved by the angle VEP, which motion is *direct*, being the same way as the apparent motion of the sun. And because the earth appears at rest with respect to the fixed stars, the planet will appear to have moved forward among the fixed stars by the angle VEP = EVT = the motion of the earth, as seen from the planet supposed at rest. Thus the planet, being on the same side of the line of direction of the earth's motion as the sun, will appear, as far as the earth's motion only is concerned, to move direct. Let M be a planet anywhere on the opposite side of the line of direction, then the planet will appear to move *retrograde* (or in a direction round E as centre opposite to that in which S appears to move) by the angle MER. And, therefore, as far as the motion of the earth only is concerned, a planet, when the line of direction of the earth's motion is between the sun and the planet, will appear retrograde.

98. To return to the apparent motion of the inferior planets. Let the earth be at E (Fig. 14), and draw two tangents GE and ED. Then when the planet is at D or G, it is at its greatest elongation from the sun S. It is clear that the planet being in the inferior part of its orbit between D and G, relatively to the earth, and the earth being supposed at rest, the planet will appear to move from left to right—that is, retrograde; and in the upper part of the orbit from right to left—that is, direct. But the earth not being at rest, we are to consider the effect of its motion. In the case of an inferior planet, the planet and the sun are always on the same side of the line of direction of the earth's motion; and therefore the effect of the earth's motion is always to give an apparent direct motion to the planet (Art. 97). Hence in the upper part of the orbit between the greatest elongations, the planet's motion will appear direct, both on account of the earth's motion, and its own motion. In the inferior part of the orbit the planet's motion will only be direct, between

the greatest elongation and the points where the retrograde motion from the planet's motion becomes equal to the direct motion from the earth's motion. At these points the planet appears stationary; and between these points, through inferior conjunction, it appears retrograde, because the retrograde motion from the planet's motion exceeds the direct motion from that of the Earth.

99. Next, for the *superior* planets, or those planets which are farther from the sun than the earth is, such as Mars, Jupiter, Saturn, Uranus, and Neptune. The interval of time between two succeeding oppositions of a superior planet to the sun can be observed. A superior planet is in *opposition* when the earth is between the sun and the planet. It is known when a superior planet is in opposition, by observing when it is in the part of the zodiac opposite to the place of the sun. Let T represent the time between two successive oppositions, then viewing the planet from the sun, the earth will appear to have gained an entire revolution, or 360° on the planet, in the time T, since the time of the earth's revolution round the sun is, in this case, shorter than that of the planet; and the earth and planet being supposed to move with *uniform* angular velocities about the sun, the angle gained by the earth will increase uniformly.

100 Let TEL (Fig. 16) represent the orbit of the earth, CDOG that of a superior planet: N the place of the planet when the earth is at E. Then, in the triangle SNE, we have the angle SEN by observation, and the angle NSE by computation. For NSE is the angle at the sun which the earth has gained on the planet, as seen from the sun, since the preceding opposition. This angle : 360° : : time since opposition : T (since the angles gained are proportional

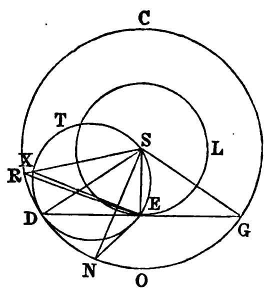

Fig. 16.

to the times of gaining them). The two angles NSE and SEN being known, the angle SNE is known, and therefore the *ratio* of SN to SE. (For sin SNE : sin SEN : : SE : SN); and consequently SN, since SE is known. Having thus obtained the distance of a superior planet from the sun, we can, *at any other time*, by help of the time T, and time of preceding opposition, compute the angular distance of the earth from the planet, as seen from the sun, and thence, by help of the earth's distance and planet's distance from the sun, we can compute the planet's elongation from the sun. Thus the planet being at R, and the earth at E, we compute the angle RSE, and knowing the sides ES and SR, and the included angle, we can compute the base angle RES—that is, the elongation of the planet from the sun. This *computed* angle being compared with the *observed* angle, we *always* find them nearly agreeing, and thereby is shown that the motions of the superior planets are accounted for by those planets being supposed to move in orbits nearly circular about the sun. As the computed place nearly agrees with the observed place, it necessarily follows that the retrograde and direct motions, and the stations, of these planets are explained, by assigning to them these circular motions.

101. And it is easy to demonstrate these appearances. It is clear that the superior planet being in any part of its orbit, and the earth being supposed at rest at any point E, the planet will appear to move from west to east, or direct. But the earth not being at rest, we are to consider the effect of its motion. The earth being at E, draw the tangent DEG; then if the planet is in the upper part of the orbit DCG, it is on the same side of the line of direction of the earth's motion as the sun, and therefore the effect of the earth's motion is to give an apparent direct motion to the planet. The earth being at E, and the planet at D or G, the planet is said to be in quadrature; consequently from quadrature to conjunction, and from conjunction to quadrature, the planet appears to move

direct, both on account of its own motion and the motion of the earth. If the planet is in the lower part of the orbit DOG, the effect of the earth's motion is to give an apparent retrograde motion to the planet; consequently from quadrature to opposition, and from opposition to quadrature, the planet moves direct or retrograde, according as the effect of the planet's motion exceeds, or is less than, the effect of the earth's motion. Between quadrature and opposition their effects become equal, and the planet appears stationary, and afterwards through opposition to the next station retrograde.

102. The apparently irregular motions of the planets among the fixed stars, must strike the most cursory observer, and it would not at first be expected that these motions could be explained by so simple an arrangement of the bodies. But it is not enough to establish the true arrangement and true motions of the bodies, that the *general* appearances are explained. It is necessary that the most minute circumstances of their apparent motions can be shown to arise from that arrangement. We have supposed above that the orbits are accurately circular, that the planes of these orbits and that of the earth coincide, and that the angular motions were uniform; but if the planes of the orbits coincided, if the orbits were accurately circular, and were uniformly described, the planets would always appear in the ecliptic, and would always be found exactly in the places which the computation on the circular hypothesis points out; but none of these things take place exactly. The deviation, however, can be explained, by showing that the planes of the orbits of the planets are inclined to the plane of the earth's orbit at small angles, and that the orbits are not circles, but only nearly circles, being ellipses, not differing much from circles, as will be shown farther on. Every phenomenon, even the most minute, can be deduced from such an arrangement; no doubt therefore would remain of the motions of the planets, in such orbits, round the sun, even had we not the evidence derived from physical astronomy.

The ancient arrangement, known by the name of the Ptolemaic system, will explain the general appearances of the planetary motions, will show when they are direct, stationary, and retrograde, and will enable us to compute nearly their apparent places; but when applied to the more minute circumstances of their motions, it totally fails.

103. The *periodic times* of the inferior planets can be deduced nearly, from observing the time between two conjunctions, their orbits being supposed circular, and the motions uniform.

Let T = the time between two successive inferior or superior conjunctions.

E = periodic time of the earth.

P = periodic time of the planet.

The angle described by the planet round the sun in the unit of time is $\frac{360°}{P}$, and that by the earth $\frac{360°}{E}$, hence their separation in this time is $\frac{360°}{P} - \frac{360°}{E}$; but since they separate by 360° in the time T, their separation in the unit of time is also $\frac{360°}{T}$: equating these quantities, and dividing by 360°, we have $\frac{1}{P} - \frac{1}{E} = \frac{1}{T}$, whenee $P = \frac{TE}{T + F}$; consequently knowing the time between two inferior conjunctions, which can be readily observed, we obtain the periodic times of the planets Mercury and Venus.

The interval between the inferior conjunctions of Mercury is 115.877 days, therefore its periodic time = $\frac{115.877 \times 365.25}{115.877 + 365.25} = 87.969$ days, or very nearly three months.

The interval for Venus is 584 days, and consequently its periodic time $= \frac{584 \times 365.25}{584 + 365.25} = 224.7$ days, about 7½ months.

104. The periodic times also of the *superior* planets can be obtained, from observing the time between two successive oppositions.

Let T, E, and P represent as before. In this case the equation becomes $\frac{360°}{E} - \frac{360°}{P} = \frac{360°}{T}$, or $P = \frac{TE}{T-E}$.

The interval between two oppositions of Uranus is $369\frac{3}{4}$ days; hence the periodic time of Uranus = $\frac{369.75 \times 365.25}{4,4} = 12 \times 365\frac{1}{4} = 82$ years. For Saturn, the interval is 387 days, and consequently the periodic time of Saturn = $\frac{378 \times 365\frac{1}{4}}{378 - 365\frac{1}{4}} = 29\frac{1}{2} \times 365\frac{1}{4} = 29\frac{1}{2}$ years. In like manner the periodic times of the other superior planets may be nearly determined.

105. The inclinations of the planes of the orbits of all the planets, except Pallas, to the plane of the earth's orbit are small. The method of ascertaining the inclinations will be afterwards shown. The points, in which a planet's orbit intersects the plane of the earth's orbit, are called *nodes.* The node through which the planet passes from the southern to the northern side of the ecliptic, is called the *ascending node,* and the other the *descending node.*

When an inferior planet is near one of its nodes at inferior conjunction, it appears a dark spot on the sun's surface, thereby is shown that the inferior planets receive their light from the sun. When Venus is in superior conjunction, at a considerable distance from its node, it may be seen, by help of a telescope, to exhibit an entire circular disc. Indeed, all the different appearances of the inferior planets, as seen through a telescope, are consistent with their being opaque bodies, illuminated by and moving about the sun in orbits nearly circular. Near inferior conjunction they appear crescents, exhibiting the same appearance as the moon a few days old. At the greatest elongation they appear like the moon when halved, and between the greatest elongation and superior con-

junction they appear gibbous, or like the moon between being halved and full.

106. These appearances are easily explained. The planet being a spherical body, the hemisphere turned towards the sun is illuminated. A small part only of this hemisphere is turned towards the earth, when the planet is near the inferior conjunction. Half the enlightened hemisphere is turned towards the earth, when the planet is at its greatest elongation. More than half, when the planet is between its greatest elongation and superior conjunction.

THEOREM.—*The greatest breadth of the part of the illumined hemisphere turned towards the earth, is proportional to the exterior angle at the planet, formed by lines drawn from the planet to the sun and earth.* Let PS (Fig. 17) be in the direction of the sun, PE in that of the earth, IPHLO the section of the planet in the plane of the earth's orbit. Draw HO perpendicular to EP, and HIO is the greatest breadth of the hemisphere turned towards the earth; IL being perpendicular to SP, IHL is the greatest breadth of the illuminated hemisphere; and HI, common to each, is the greatest breadth of the illuminated part seen from the earth. The measure of this is the angle IPH = IPS + SPH = HPG+SPH=SPG the exterior angle at the planet. Now near inferior conjunction the exterior angle is less than a right angle; at the greatest elongation it is a right angle; and afterwards greater than a right angle. Therefore the breadth of the illuminated part is respectively less than a quadrant, equal to a quadrant, and greater than a quadrant.

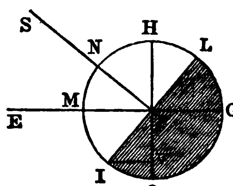

Fig. 17.

107. It is easy to see that as the planets appear flat discs on the concave surface, so their illumined parts will be projected on the flat surface, and the greatest breadth HNMI will be projected into its versed sine

HV. Because the projection of a circle, inclined to a surface, by right lines perpendicular to that surface, is an ellipse, the inner termination of the enlightened part appears elliptical, and the enlightened surface: surface of planet : : HV, the projection of greatest breadth of illuminated surface HI, : the diameter HO, or : : versed sine of exterior angle : diameter.

108. With respect to the superior planets; the exterior angle of the planet is least when the planet is in quadrature. For when the exterior is least the interior is greatest. Now it is evident that SDE (Fig. 16), when DE is a tangent to the orbit of the earth, is greater than when E is at any other point, R; for on the radius SD as diameter, let a semicircle be described, the angle ERS is less than EXS, which is equal to EDS, being in the same segment, consequently the exterior angle in the former case is greater than in the latter, and therefore the planet being in quadrature, the exterior angle is least. SDE for every superior planet is acute, and the exterior angle obtuse, and consequently its versed sine is greater than radius. Whence more than half the disc of a superior planet is always seen, and it appears most gibbous in quadrature. Mars then appears gibbous about $\frac{1}{8}$ of his diameter; Jupiter only by about $\frac{1}{100}$ of his diameter, which quantity is imperceptible, even by a telescope; because Jupiter's disc then only subtends an angle of 30″. Accordingly all the superior planets, except Mars, appear always with a full face. The asteroids appear so small, that it cannot be expected that they should appear in any degree gibbous.

109. The brightness of a planet depends both on the quantity of illuminated surface and its distance. The greater the distance is, the less the brightness; which, the illuminated surface remaining the same, decreases as the square of the distance increases, so that in computing when a planet appears brightest, both the illuminated surface and distance must be taken into account. Both circumstances concur in making a superior planet appear brightest at opposition. The inferior planets

are not brightest at superior conjunction, because of their greater distance; and near inferior conjunction the illuminated part visible to us is very small. The place of greatest brightness then lies between inferior and superior conjunction.

The solution of the problem to find when Venus appears brightest, gives her elongation then about 40 degrees. The places of greatest brightness are between the places of greatest elongation and inferior conjunction. This agrees very well with observation. When she is near this position she occasions a strong shadow in the absence of the sun; and for a considerable time both before and after she is at this elongation, she may be readily seen in full day-light by the naked eye.

110. From inferior to superior conjunction Venus is to the westward of the sun, and therefore rises before the sun, and by the splendour of her appearance, being much noticed, is called the morning star. From superior to inferior conjunction she appears to the eastward of the sun, and therefore does not set till after the sun, and is then called the evening star. Jupiter, which approaches much nearer in splendour to Venus than any other planet, is sometimes called a morning or evening star, according as it rises before or sets after the sun, and when near opposition may be called both an evening and morning star. Since the greatest elongation of Mercury from the sun is about $22\frac{1}{2}°$, it is never seen except during twilight, and consequently does not attract much attention when seen by the naked eye. In order to observe this planet most satisfactorily, it should be at its greatest distance from the sun when at its greatest elongation, twilight should also be shortest at the same time, and the polar distance of Mercury should be also much less than that of the sun, in order that the difference of the times of rising or setting of the sun and planet should be the greatest possible.

111. *The following Tables exhibits at one view the principal outlines of the Planetary System.*

	Merc. ☿	Ven. ♀	Earth ⊖	Mars. ♂	Jup. ♃	Sat. ♄	Uranus ♅	Sun. ⊙	Nep.
Mean distances from sun, earth's dist. being 10.	4	7	10	15	52	95	192		300
Periodic time.	days 88	days 224	days 365	days 686	years 12	years 29½	years 83		years 165
Diameter, earth's diam. 10.	4	9	10	8	110	100	43	1128	44
Inclination of orbit to ecliptic.	° 7	° ′ 3 23		° ′ 1 51	° ′ 1 19	° ′ 2 30	° ′ 0 46		° ′ 1 47
Place of ascending node as seen from sun.	° 45	° 74		° 46	° 97	° 111	° 73		° 130 8
Diameter of sun seen from planet.	′ 80	′ 46	′ 32	′ 21	′ 6	′ 3	1		
Time of revolution on axes.		h m 23 21	h m 23 56	h m 24 37	h m 9 55	h m 10 29		d h 25 10	
Days from conj. to conj. or opp. to opp.	115	584		780	399	378	369½		367½
Of which time they retrograde, during days.	22	42		70	120	135	151		
Arcs which they retrograde.	° 12	° 16		° 18	° 9	° 6	° 4		
Velocity per second in miles.	30	23	19	15	8	6	4⅓		3½
Greatest and least apparent diam.	″ ″ 11 5	″ ″ 57 10		″ ″ 26 5	″ ″ 40 26	″ ″ 18 15	″ 4		″ 2.6

	Mean distance.	Periodic time in days.	Inclination of orbit to ecliptic.	Diameter of sun as seen from planet.
Ceres, . . .	28	1681	10°27′	11′
Pallas, . . .	28	1683	34°39′	11′
Juno, ...	27	1591	13° 5′	12′
Astræa, . . .	26	1510		12′
Vesta, . . .	24	1325	7° 9′	13′

The times and arcs of retrogradation are computed on the supposition that the orbits are circular.

The apparent diameters of the asteroids have not been ascertained. They are too small to be measured by micrometers.

Sir W. Herschel thinks that if the diameter of any one of them amounted to $\frac{1}{4}$th of a second, he should have been able to have ascertained it.—Phil. Tran., Part 1, 1805.

112. Perhaps the most striking circumstance in the above table, is the great velocities with which the planets move; and this is more impressed, when we consider that of the earth on which we live, the velocity of which is 90 times greater than the velocity of sound. In contemplating these velocities, it cannot but occur to us how great a power is necessary to be continually acting, to circumflect the planets about the sun, and compel them to leave the tangential direction (Art. 79). A power that acts incessantly, and is able to counteract the great velocities of the planets must excite our inquiries as to its origin and law of action.

We can ascertain that this power is constantly directed towards the sun, increases in intensity as the square of the distance from the sun decreases, and that it is the same power which is diffused through the whole planetary system, only varying in quantity as

the square of the distance from the sun is varied. So far physical astronomy teaches us; but the proximate cause of this power, or solar gravity, as it may be called, is unknown. We cannot trace by what agency the Supreme Being, from whom all things originate, has ordained the operations and laws of gravity to be executed.

113. By a comparison of the distances and periodic times, which are determined independently of each other, it will be seen that *the squares of the periodic times* are to each other as the *cubes of the distances from the sun.* This relation was first found out by Kepler. For a long time no necessary connexion was discovered between the periodic times and distances, till at last it was shown by Newton to be a consequence of the law of gravity above-mentioned.

114. At present we know of no secondary cause that could have any influence in regulating the respective distances of the planets from the sun; yet there appears a relation between the distances, that cannot be considered as accidental. This was first observed by Professor Bode of Berlin, who remarked that a planet was wanting, at the distance at which the asteroids have since been discovered, in order to complete the relation. According to him, the distance of the planets may be expressed nearly as follows, the earth's distance from the sun being 10.

Mercury	4	=	4
Venus	$4+3\times 2^0$	=	7
Earth	$4+3\times 2^1$	=	10
Mars	$4+3\times 2^2$	=	16
New planets	$4+3\times 2^3$	=	28
Jupiter	$4+3\times 2^4$	=	52
Saturn	$4+3\times 2^5$	=	100
Georgium Sidus	$4+3\times 2^6$	=	196
Neptune	$4+3\times 2^7$	=	388

Comparing these with the mean distances above given, we cannot but remark their near agreement, and can scarcely hesitate to pronounce that these mean

distances were assigned according to a law, although we are entirely ignorant of the exact law and of the reason for that law.[a]

115. Astronomy must have been considerably advanced before any attempts were made to ascertain the position of the planets with respect to the sun and to each other, and to develope their motions. It is said, however, that the Egyptians very early conceived the motions of the planets Mercury and Venus to be about the sun, and also that the Pythagoreans considered the sun as the centre about which the planets performed their motions. But their opinions are so imperfectly expressed in the few scattered notices which are found in different authors, that little can be known with certainty about them.

The distinguished astronomers of the Alexandrian school, Aristarchus, Eratosthenes, Hipparchus, and others, seem not to have attempted any theory of the planetary motions, notwithstanding they far excelled in other parts of astronomical knowledge all that had gone before. And we are certain that till Ptolemy, who wrote about 140 years after the birth of Christ, published the system that goes by his name, the motions of the planets were not submitted to regular calculation.

In the Ptolemaic system, the earth is supposed immoveable in the centre about which the Moon, Mercury, Venus, the Sun, Mars, Jupiter, and Saturn are supposed to revolve in different periods and in the order stated. All these bodies, as well as the fixed stars, were likewise supposed to be carried round the earth by the motion of the *primum mobile* in 24 hours. The latter opinion appears now so unphilosophical, that we are apt to judge by it of the rest, and despise the whole

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[a] The deviation of this empirical law in the case of Neptune is very considerable. The number corresponding to the new planets has also no meaning for the asteroids, and at least represents only the *average* of their distances from the sun. This is, for instance, for Flora 22, and for Hygeia 31.

Ptolemaic system, as unworthy of consideration. However, that part of the system by which the inequalities of the planetary motions were explained is well worthy of examination, and seems in some measure entitled to the credit which it possessed for nearly fourteen centuries.

NOTE BY THE EDITOR.—By means of Kepler's third law, which connects the periodic times and the distances of the planets from the sun, we can find the ratio of their velocities in this manner: Let v be the velocity of the planet nearer to the sun, and v' that of the farther planet, T and T$'$ their periodic times, then if r and r' be the distances from the sun, $T^2 : T'^2 :: r^3 : r'^3$, but $\left(\text{since } T = \frac{4\pi r}{v}\right)$ $T^2 : T'^2 :: \frac{r^2}{v^2} : \frac{r'^2}{v'^2}$. Therefore $\frac{r^2}{v^2} : \frac{r'^2}{v'^2} :: r^3 : r'^3$, or $\frac{r'}{v^2} = \frac{r}{v'^2}$, that is, $v : v' :: \sqrt{r'} : \sqrt{r}$.

Hence may be determined the relative apparent motion of a planet with regard to the sun. If the planet be an inferior one, such as Mercury or Venus, at inferior conjunction the space described in a day by the planet is, from the above proportion, greater than that described by the earth, and as both motions are then parallel, the planet seems to move away from the sun, or its motion is retrograde. To find the stationary point it is evident that for two consecutive days the lines drawn from the earth to the planet must be drawn parallel in space. If S be the sun and P and P$'$ two consecutive positions of the planet in its orbit, and E and E$'$ two consecutive positions of the earth, the line PE must be parallel to P$'$E$'$; hence the perpendicular E$'p$ on EP must be equal to the perpendicular Pp' on E$'$P$'$. If the angle PES be called θ, and SPE θ', and V and V$'$ the spaces, PP$'$ and EE$'$, described in one day by Venus and the earth, it is evident that these perpendiculars are V $\cos\theta$ and V$'$ $\cos\theta'$ (since the motions are small and perpendicular to the distances SE, SP $\therefore$ V : V$'$ :: $\cos\theta' : \cos\theta$, or $\sqrt{SP} : \sqrt{SE} :: \cos\theta' : \cos\theta$ .... (a).

But SP : SE : : $\sin\theta : \sin\theta' \ldots (\beta)$. Hence eliminating θ' from these two equations, we get from (a) $\cos\theta' = \frac{\sqrt{SP}}{\sqrt{SE}}\cos\theta$, and from ($\beta$) $\sin\theta' = \frac{SE}{SP}\sin\theta$; squaring and adding we have $\frac{SP}{SE}\cos^2\theta + \frac{SE^2}{SP^2}\sin^2\theta = 1$; or solving, $\sin\theta = \frac{SP^2}{SE^2 + SE \times SP + SP^2}$. If we divide $\sin^2\theta'$ by $\cos^2\theta'$ we have $\tan^2\theta' = \frac{SE^3}{SP^3}\tan^2\theta$, or (since $SE^3 : SP^3 : : E^2 : P^2$, E being the earth's period, and P the planet's) $\tan\theta' : \tan\theta : : E : P$, or PB : EB : : P : E (B being the foot of a perpendicular from S on EP). From this proportion we may find graphically the stationary point of a planet in the following manner:—Draw two concentric circles representing the orbits of the planet, and of the earth round the sun, join the centre S with the place of the earth E, cut from SE a part SX, which shall be : SE as P : E; on EX describe a semicircle, this will cut the planet's orbit at the stationary point. This construction is due to Professor Mac Cullagh.

CHAPTER VIII.

ON THE SECONDARY PLANETS AND MOON—ATMOSPHERES OF PLANETS—RINGS OF SATURN—COMETS—METEORS.

116. JUPITER'S SATELLITES.—Four small stars, only visible by the help of telescopes, always accompany Jupiter, and are continually changing their positions with respect to each other and Jupiter. They are called satellites and secondary planets. The first satellite is that which elongates itself least from Jupiter, &c. They clearly show that Jupiter is an opaque body enlightened by the sun; for when they intervene between him and the sun, they project a shadow on his disc, which appears to move across it like a minute dark spot. They themselves are also opaque bodies illuminated by the sun; for when the planet intervenes between any of them and the sun, they are eclipsed. The phenomena prove that they revolve about Jupiter at different distances in orbits nearly circular, while they are carried together with him in his orbit round the sun. Their orbits are inclined to the plane of Jupiter's orbit, as is concluded from the unequal durations of the eclipses of the same satellite, for if their orbits coincided with that of Jupiter each would always take the same time in passing through the same space, the diameter of the section of the shadow. The fourth satellite is in extremely rare cases in opposition to the sun, without being eclipsed. This is owing to the inclination of its orbit to that of Jupiter, which enables it to pass over the shadow without being immersed in it at opposition. The third and fourth satellites disappear and re-appear on the same side of Jupiter. Only

the beginnings or the endings of the eclipses of the first and second satellite are visible, since their orbits are so close to Jupiter that the body of the planet intervenes and prevents our seeing the egress from the shadow if the ingress is visible, and *vice versâ.*

117. Let S (Fig. 21) be the sun; J Jupiter and its shadow; E and E′ the earth before and after the opposition of Jupiter; AB the path of the first satellite in the shadow; E*t* a tangent to Jupiter. When the first satellite enters the shadow, the apparent distance of the satellite from the body of Jupiter is AET; but at its emersion at B, the line EB always passes through Jupiter, and therefore the emersion is invisible; but after opposition, the earth being at E′, the emersion and not the immersion will be visible. The same things take place with respect to the second satellite. If CD be the path of the third satellite, DE frequently lies without the body of Jupiter, and therefore both the immersion at C and the emersion at D are visible; and the phenomena are very striking, from the circumstances of the satellite disappearing and re-appearing at a distance from the body of Jupiter on the same side. The same may be observed with respect to the fourth satellite. Before the opposition of Jupiter to the sun, the eclipses happen on the west side of Jupiter, as in the figure, as Jupiter is to the east of the satellite, as seen from E; after opposition, on the east. If the telescope invert, the contrary takes place.

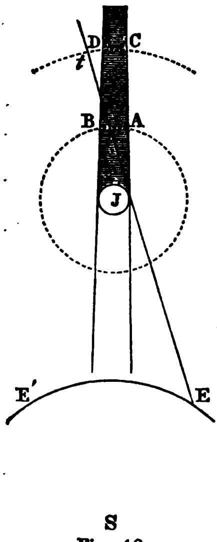

Fig. 18.

In addition to the *eclipses* of Jupiter's satellites, and the *transits of their shadows* over his disc, the satellites

sometimes pass behind Jupiter, as seen from the earth, and then there is an *occultation* of the satellite by the planet, and sometimes the satellite itself passes between the earth and Jupiter, when there is a *transit* of the *satellite*, which appears sometimes brighter and sometimes darker than the portion of Jupiter's surface which it crosses. When the planet is in quadrature all these phenomena may be witnessed.

118. It has long been suspected that the satellites of Jupiter revolve on their axes; and Sir W. Herschel has supposed that each of them revolves in the time of its revolution round the primary. This is inferred from the periodical changes in their brightness corresponding to the period of their revolutions round Jupiter. Their motions about the primary, and their motions about their axes, are from west to east.

119. Their distances in miles from Jupiter and their periodic times are nearly as follow:—

Sat.	Dist.	Per.	Sat.	Dist.	Per.
I.	280,000	$1^d\ 18^h$	III.	700,000	$7^d\ 4^h$
II.	440,000	$3^d\ 13^h$	IV.	1,200,000	$16^d\ 16^h$

The distance of the fourth satellite is only 13 times the diameter of Jupiter, or 8′ 18″ as seen from the earth, or about one-fourth the apparent diameter of the moon.

They must be very magnificent objects to the inhabitants of Jupiter. The first satellite appears to them with a disc four times greater than that of our moon appears to us, and goes through all the changes of our moon in the short space of 42 hours, within that period being itself eclipsed, and causing an eclipse of the sun on the surface of Jupiter.[a]

120. The order of their magnitude is 3^d, 4^{th}, 1^{st}, 2^d, according to Sir W. Herschel. Their masses, that of the earth being 1000, and therefore of the moon 14, are

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[a] The mean angular velocity of the first satellite + twice that of the third = three times that of the second; from this it follows that if their mean longitudes as seen from Jupiter be l_1, l_2, l_3; $l_1 + 2l_3 - 3l_2$ = a constant quantity, which is found by observation to be 180° Hence it may be easily shown that they cannot be all eclipsed together.

Sat.	Mass.	Sat.	Mass.
I.	5	III.	27
II.	7	IV.	13

121. The satellites of Jupiter, at their greatest elongations, appear nearly in the direction of the equator of Jupiter, because the equator of Jupiter and the orbits of the satellites are inclined, at small angles, to the plane of Jupiter's orbit. The direction of Jupiter's equator is marked by the *belts* of Jupiter, which are faint shades, parallel to each other, on the body of Jupiter, and which frequently undergo such changes, that they have been supposed to be somewhat of the nature of clouds in his atmosphere; but, from some unknown cause, more permanent than our clouds. This is supposed to arise from strong and constant currents in the atmosphere of Jupiter, in the direction of his equator, of a similar nature to our trade winds, and produced by the same cause.

122. Galileo discovered the four satellites of Jupiter, Jan. 7, 1610. This, which might naturally have been a source of delight, was at first a subject of disappointment. He supposed them to be fixed stars, and found, looking at them on the next night, that Jupiter was to the eastward of them, whence he concluded the motion of Jupiter direct; whereas, according to the Copernican system, it ought then to have been retrograde; but he soon discovered that the motion was in what he took for fixed stars, and announced his discovery to the world. Harriot also appears to have discovered them about the same time that Galileo did.

This discovery was very important in its consequences. It furnished, as we shall see, a ready method of finding the longitude of places by means of the eclipses of the satellites which so frequently take place. This made the eclipses be particularly attended to, which led Roemer to discover that the transmission of light is not instantaneous; and this led Bradley to account for a small apparent motion of the fixed stars, called the aberration of light, which has furnished an

independent proof of the motion of the earth, as strong as that from physical considerations.

123. SATURN'S SATELLITES.—Saturn has eight satellites revolving about him in orbits nearly circular, of which the sixth is seen without much difficulty, and was called the Huygenian satellite, from having been discovered by Huygens in 1655. The third, fourth, fifth, seventh, and eighth were afterwards discovered. Sir W. Herschel discovered the first and second.

It has long been supposed that the seventh (formerly the fifth) satellite revolved on its axis in the time of its revolution round Saturn. This has been confirmed by the observations of Sir W. Herschel. These satellites, except the sixth, require a very good telescope to render them visible, on which account they have been much less attended to than the satellites of Jupiter. The distances from Saturn in semi-diameters of Saturn, and periodic times, are nearly as follow. The equatorial semi-diameter of Saturn is 37,500 miles:—

Sat.	Dist.	Per.	Sat.	Dist.	Per.
I.	3·36	$0^d\ 22^h\ 37^m$	V.	9·55	$4^d\ 12^h\ 25^m$
II.	4·31	$1^d\ 8^h\ 53^m$	VI.	22·14	$15^d\ 22^h\ 41^m$
III.	5·34	$1^d\ 21^h\ 18^m$	VII.	28	$22^d\ 12^h$
IV.	6·84	$2^d\ 17^h\ 41^m$	VIII.	64·36	$79^d\ 7^h$

124. Sir W. Herschel in 1787 discovered two satellites to Uranus. Their orbits are nearly perpendicular to the orbit of their primary. Two others have been since observed.

The relation of the periodic times, and distances of the satellites from their primary, holds in all the secondaries of each planet respectively.

125. THE MOON.—Next to the sun, the most interesting to us of all the celestial bodies, is our own satellite, the moon. It apparently describes, by a motion from west to east, on the concave surface of the celestial sphere, a great circle, nearly intersecting the ecliptic at an angle of about 5°. This apparent motion is explained by a real motion round the earth, in an orbit inclined to that of the earth, at an angle of 5°. The periodic

time, or time of return to the same point of the concave surface, or the same fixed star is 27 d. 7 h. 43 m. The variation of diameter shows the variation of distance is greater than the variation of the sun's distance. The greatest diameter is $33'\frac{1}{2}$, least $29'\frac{1}{2}$, and the mean $31'\frac{1}{2}$. The moon is carried with the earth in its annual motion round the sun. This necessarily follows, if the motion of the earth be granted, and is well illustrated by the motion of the satellites of Jupiter and Saturn. The apparent motion of the moon on the celestial concave surface varies considerably from its mean quantity, and its variations seem very irregular. Its greatest hourly motion in its great circle is 33′ 40″, its least 27′, mean 32′ 56″; so that in its mean quantity it moves over an arc of the heavens equal to its apparent diameter in about an hour.

126. The intersections of its apparent path with the ecliptic, or the intersections of its orbit, and the earth's orbit, called its *nodes*, are not fixed, but move backward, at the rate of about 19° in a year, completing a revolution in 6794 days = 18 years 224 days. If we conceive then a circle inclined to the ecliptic, at an angle of 5 degrees, and a body moving in this circle at the rate of about 33′ in an hour, while the circle itself is carried backward with a slow motion of 8′ an hour, the path of this body on the concave surface will in some measure represent the path of the moon. The more accurate considerations of the lunar motions will be resumed hereafter. The full investigation of the motions of the moon is one of the most intricate, and, as connected with finding the longitude at sea, one of the most useful problems in astronomy. Perhaps in no instance has modern science reaped so much credit as from the success that has followed the attempt to completely develope the lunar motion.

127. MOON'S PHASES.—The phases of the moon are particularly interesting; they prove the moon to be a spherical body illumined by the sun. When in conjunction with the sun, the moon is invisible, inasmuch

as the illuminated hemisphere is turned away from the earth; when, moving from the sun towards the east, it is first visible, it is called the new moon, and appears a crescent: when in quadrature, or 90 degrees from the sun, it is halved: when more distant, it is gibbous, and when in opposition, it shines with a full face, the whole of the illuminated surface being then turned towards us; approaching the sun towards the east, it becomes again gibbous, then halved, and lastly a crescent, after which it disappears, from the superior lustre of the sun, and the smallness of the illuminated part which is turned towards the earth.

128. The enlightened part varies nearly as the versed sine of the angle of elongation from the sun. It is proved in the same manner[a] as for the planets, that the enlightened part varies as the versed sine of the exterior angle at the moon, formed by lines drawn from the earth and sun to the moon. But this exterior angle is equal to the angle of elongation + angle subtended at the sun by the distance of the earth from the moon (Euclid, I., 32). The latter angle never amounts to $10'$, and therefore is inconsiderable.

129. Moon's Periodic Time.—The time between two conjunctions or two oppositions called a *lunation,* and *synodic month,* is greater than the time of a revolution in the orbit, or the time of return to the same fixed star. Because, when the moon has come back to the same point of the heavens she has to move a farther space to overtake the sun, which has moved on in the ecliptic through $17°$.

To find the moon's siderial period.

Let S = period of sun's apparent motion about the earth.

P = period of moon's motion about the earth.

L = period between conjunction and conjunction, or of a lunation.

There the angle described by the moon round the earth in the unit of time is $\frac{360°}{P}$, and the angle described by

[a] Art. 106 and 107.

the same time is $\frac{360°}{S}$; therefore the separation between them in the unit of time is $\frac{360°}{P} - \frac{360°}{S}$, but this is also $\frac{360°}{L}$, equating these we have $\frac{360°}{P} - \frac{360°}{S} = \frac{360°}{L}$; hence $\frac{1}{P} - \frac{1}{S} = \frac{1}{L}$, therefore

$$P = \frac{S \times L}{S + L} = \frac{365.25 \times 29.5305887}{394.7805887}$$

= 27 days, 7 hours, 40 minutes nearly. The mean synodic period may be determined by the comparison of very distant eclipses of the moon.

130. In 19 solar years of $365\frac{1}{4}$ days, there are 235 lunations and 1 hour, for $29{\cdot}5305887 \times 235 = 6939.688$, and $365\frac{1}{4} \times 19 = 6939{\cdot}75$. Therefore, considering only the mean motion, at the end of 19 years, the sun, moon, and earth, return to the same relative positions with regard to the fixed stars, and the full moons fall again upon the same days of the month, and only one hour sooner. This is called the Metonic Cycle, from Meton, who published it at the Olympic Games, in the year 433 B. C. This period of 19 years has been always in much estimation for its use in forming the calendar, and in calculating the time of Easter, which depends on the time of the first full moon after March 21st; and from that circumstance the numbers of this cycle of years have been called the *golden numbers.*

131. The appearance observed a few days before and after the new moon, when the moon is a crescent, namely that the remainder of the moon's surface appears to shine with a pale grey light, is caused by the strong earth-light which then shines upon the moon, or by the reflection of the sun's rays from the earth to the moon, which are reflected back again to the earth. When the moon becomes considerably elongated from the sun, very little of this light falls on the moon. The pale grey portion of the moon appears to belong to a smaller circle than the bright crescent does. This phenomenon

affords a remarkable proof, that of two objects of the same magnitude, the brighter object appears larger by an optical illusion.

132. One of the earliest attempts upon record to discover the distance of the sun from the earth, was from observing when the moon was exactly halved or dichotomised. At that time the angle at the moon, formed by lines drawn from the moon to the sun and earth, is exactly a right angle; therefore if the elongation (E) of the moon from the sun be exactly observed, the distance of the sun from the earth will be had, that of the moon being known, by the solution of a right angled triangle, that is, sun's distance = moon's distence × sec E. The uncertainty in observing when the moon was exactly dichotomised, rendered this method of little value to the ancients. However, by the assistance of micrometers, it may be performed with considerable accuracy. Vendelinus, observing at Majorca, the climate of which was well adapted to observation, determined in 1650, the sun's distance, by this method, very considerably nearer than had been done at that time by any other method.

This method is particularly worthy of attention, being the first attempt for the solution of the important problem of finding the sun's distance. It was used by Aristarchus of Samos, who observed at Alexandria, about 280 years before the commencement of the Christian era. He found that the line which divided the light and darkness passed through the centre of the moon when her elongation was 87°, hence he inferred that the sun's distance was only 19 times greater than the moon's distance; it is in reality 385 times that distance.

133. Viewing the moon with a telescope, several curious phenomena offer themselves. Great variety is exhibited on her disc. There are spots differing very considerably in degrees of brightness. Some are almost dark. Many of the dark spots must necessarily be excavations on the surface, or valleys between mountains, from the circumstances of the shades of light which they exhibit. There is no reason to suppose that there is any

large collection of water in the moon; for if there were, when the boundary of light and darkness passes through it, it must necessarily exhibit a regular curve, which is never observed; on the contrary the bounding line is generally very broken and irregular, such as would be produced by a lunar surface pitted with deep indentations and diversified by frequent elevations. The non-existence of large collections of water is also probable from the circumstance of no changes being observed on her surface, such as would be produced by vapours or clouds; for, although, as will be remarked, the atmosphere of the moon is comparatively of small extent, yet it is probable that an atmosphere does exist.

134. LUNAR MOUNTAINS.—That there are lunar mountains is strikingly apparent, by a variety of bright detached spots almost always to be seen on the dark part, near the separation of light and darkness.

These are tops of eminences enlightened by the rays of the sun passing over the circle of light and darkness, while their lower parts are still in shade. But sometimes light spots have been seen at such a distance from the bright part, that they could not arise from the light of the sun. Sir W. Herschel has particularly noticed such at two or three different times. These he supposes are volcanoes. He measured the diameter of one, and found it $= 3''$, which answers to four miles on the surface of the moon.

135. MEASUREMENT OF HEIGHTS OF LUNAR MOUNTAINS.—The heights of lunar mountains may be ascertained by measuring with a micrometer the distance between the top of the mountain, at the instant it first becomes illuminated, and the circle of light and darkness. This measurement is to be made in a direction perpendicular to the line joining the extremities of the horns, and therefore parallel to the plane of the earth, sun, and moon, because this line is perpendicular to that plane, being the intersection of two planes passing through the moon's centre, one perpendicular to the line joining the earth and moon, the other to the line joining the moon and sun.

Let ACD (Fig. 19) be the circle of light and darkness, T the top of a mountain just illumined by the ray KDTE coming in a direction perpendicular to the plane of the circle ACD, and being tangent to the surface

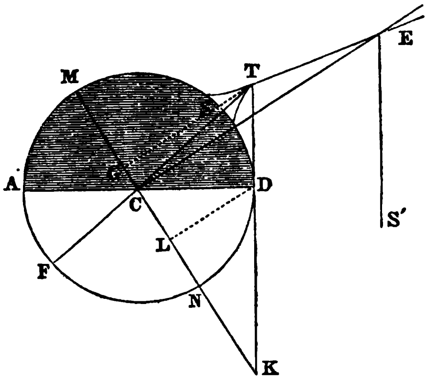

Fig. 19.

at D. Let S be at the surface, or the bottom of the mountain, and C the centre of the moon; then (by Euclid, 3 B. 36) $FT \times TS = DT^2$, or $TS \times (TS + 2CS) = DT^2$; but TS being very small compared with CS, we may omit TS^2, and we have $TS \times 2CS = DT^2$, or $TS = \frac{DT^2}{2CS}$. We cannot measure DT directly, because we observe only GL the projection of DT on the plane of the circle of vision MCN. Now DT being parallel to the plane of the circle AMD, makes an angle DKC with MN, the plane of the circle of vision, = the complement of the angle KCD. Therefore DT observed, or GL, = DT cos DKL = DT × sin KCD = DT × sin of the angle of elongation of the moon from the sun, since the angle between CK and CD is equal to the angle CES′ between two lines ES′ and EC drawn through the earth E perpendicular to CK and CD, but one of these perpendiculars will point to the sun, S′, and the other to the moon,

the angle which EC subtends at the sun being only $10'$ may be neglected. Hence $DT = \frac{\text{DT observed (GL)}}{\text{sin elong.}}$, and consequently $TS = \frac{(\text{DT observed})^2}{2\text{CS} \times \sin^2 \text{elong.}}$.

Old writers on astronomy, when mentioning this problem, have not considered that the projection of TD was only measured, and not TD itself, as has been remarked by Sir W. Herschel. Their methods, therefore, only held when the moon was elongated 90° from the sun.[a]

The height of a lunar mountain is sometimes measured by the length of the shadow which it casts

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[a] Ricciolus mentions that, on the fourth day after new moon, he observed the top of the hill, called St. Catherine's, to be illuminated, and that it was distant from the confines of the lucid part about a sixteenth part of the moon's diameter. Hence computing according to his method—that is, supposing DT itself $\frac{1}{16}$ part of the moon's diameter, and calling the moon's diameter unity, $TS = \frac{1}{16} \times \frac{1}{16} = \frac{1}{256}$ part of the moon's diameter, and as the moon's diameter = 2000 miles nearly, $TS = \frac{2000}{256} = 8$ miles nearly, the height of St. Catherine's according to Ricciolus; but on the fourth day after new moon, the moon could not be farther elongated from the sun than 48°. Therefore TS could not be less than $\frac{8 \text{ miles}}{\sin^2 48°} = \frac{8}{(.74)^2}$ $14\frac{3}{4}$ miles nearly. But later astronomers are not inclined to allow of so great an elevation to any of the lunar mountains. St. Catherine's is now set down at 16,400 feet. Sir W. Herschel investigated the heights of a great many; and he thinks that, a few excepted, they generally do not exceed half a mile. But there seems to be little doubt that there are mountains on the surface of the moon which much exceed those on the surface of our earth, taking into consideration the relative magnitudes of the moon and earth. Taking the altitude of Newton, the highest of the lunar mountains, to be 23,800 feet, and that of Mount Everest, in the Himalayas, to be 29,000 feet, considering that the moon's diameter is only $\frac{3}{11}$ of that of the earth, the lunar mountains are relatively three times higher than those of the earth.

on the moon's surface, compared with the direction of the sun's rays.

136. MOON'S LIBRATIONS.—It is not the least remarkable circumstance of the moon, that it always exhibits nearly the same face to us. We always observe nearly the same spots, and that they are always nearly in the same position with respect to the edge of the moon. Therefore as we are certain of the motion of the moon round the earth, we conclude that the moon must revolve on an axis nearly perpendicular to the plane of her orbit, in the same time that she moves round the earth, viz. in $27\frac{1}{2}$ days. This must necessarily take place in order that the same face may be continually turned towards the earth during a whole revolution in her orbit. The motion of the moon in her orbit is not equable, therefore if the rotation on her axis be equable, there must be parts in her eastern and western edges, which are only occasionally seen. These changes, called her *libration in longitude*, are found to be such as agree with an equable motion of rotation. There are parts about her poles only occasionally visible. This, called her *libration in latitude*, arises from her axis being constantly inclined to the plane of her orbit, in an angle of $83\frac{1}{2}°$, consequently the northern and southern poles of the moon are sometimes turned nearer, and sometimes farther from the earth through an angle of 13°; when one of the poles is turned towards us, we see more of the surface about that pole than when it is turned away from us. A *diurnal libration* also takes place; at rising, a part of the western edge is seen, that is invisible at setting, and the contrary takes place with respect to the eastern edge. This is occasioned by the change of place in the spectator, occasioned by the earth's rotation, bringing him at the equator in twelve hours to a distance nearly 8000 miles from his former stand point with regard to the moon.

137. A few remarks may be here made concerning the rising and setting of the moon at different seasons, and of some other circumstances of moon-light.

The rising and setting of the moon is most interest-

ing at and near full moon. At full moon, it is in or near that part of the ecliptic, opposite to the sun. Hence at full moon, at midsummer, it is in or near the most southern part of the ecliptic, and consequently appears but for a short time above the horizon; and so there is little moon-light in summer, when it would be useless. In mid-winter, at full, it is near or in the northernmost part of the ecliptic, and therefore remains long above the horizon, and the quantity of moon-light is then greatest when it is most wanted; and this is the more striking, the nearer the place is to the north pole. There at mid-winter, the moon does not set for fifteen days together—namely, from the first to the last quarter.

138. The moon, by its motion from west to east, rises later every day, but the retardations of rising are very unequal even during the same month. In northern latitudes, when the moon is near the vernal intersection of the ecliptic and equator, or the beginning of Aries, the retardation of rising is least, and when near the beginning of Libra, greatest. This will appear by considering that when the first point of Aries is on the eastern horizon, the ecliptic makes the *least* angle with the horizon (namely, the colatitude—23° 28′), and when it is on the western horizon, and consequently Libra, the opposite sign, on the eastern, the ecliptic makes the greatest angle (namely, the colatitude + 23° 28′). This is seen if we remember that the angle between two great circles is proportional to the arc between their poles, and as the pole of the ecliptic describes a diurnal circle round the pole of the equator, the distance between it and the zenith changes; it is greatest when the pole of the ecliptic is on the meridian below the pole, and least when on the meridian above the pole; in the latter case Aries is on the east horizon.

To explain this more fully, let SH′CHN (Fig. 20) represent the horizon, CL a portion of the ecliptic when the first point of Aries is at C, and ECMNQ the equator. Suppose the moon to rise at C on one night,

then after a revolution of the concave surface, the circles will come again into the same position with respect to the horizon, but the moon will have advanced, suppose to L (in this illustration we consider the moon *as moving in the ecliptic*). Let HL be a portion of a small circle parallel to the equator described in her diurnal motion, by the moon when at L, then on

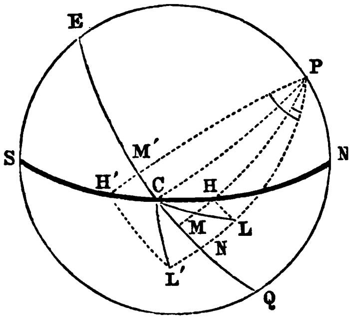

Fig. 20.

the second night the moon will rise nearly at H, and therefore, HM and LN being secondaries to the equator, ECQ, when the first point of Aries is rising at C, the moon being at L will have to traverse the space LH before she rises at H, and the time of describing LH is measured by the angle HPL at the pole P, or by the corresponding arc of the equator MN. MN + $23^h\ 56^m$ or CN − CM + $23^h\ 56^m$ will consequently be the interval elapsed between two successive risings. If CL′ be a portion of the ecliptic when the moon in Libra is rising, and L′ the place of the moon on the second night, then L′H′ being in this case the diurnal circle described by the moon, H′ will be nearly the place of rising, and the interval in time between the rising on the second night of the first point of Libra and the moon will be measured by the angle H′PL′, or the arc M′N of the equator, and consequently the interval of time between the two successive risings of the moon M′N + $23^h\ 56^m$, or CN + CM′ + $23^h\ 56^m$. It will readily appear that CM′ = CM, because LN = L′N nearly, since CL = CL′ and LCN = 23°28′ = L′CN. Hence the retardation, when the moon rises in Libra, is greater than the retardation when the moon is in Aries, by 2CM or the angle M′PM reduced to time. It is easy to see that

these are the two extremes of retardation; for the moon's retardation in rising at any time on two consecutive nights is proportional to HPL, if C be the point of the horizon where she rises on the first night, and H on the second, and CL her motion in the interval on her orbit, supposed to coincide with the ecliptic. Now the arc of the small circle HL = HPL × sin HP (since its radius is proportional to sin HP) $\therefore \text{HPL} = \frac{\text{HL}}{\sin \text{HP}}$, consequently this fraction will be the least when its numerator (HL) is the least, and its denominator sin HP the greatest. In the triangle CLH, since CL is supposed to be constant, and NHL is nearly equal to HCQ, the colatitude, HL will be the least when the angle HCL is least, and sin HP will be greatest when the moon is on the equator: both of these jointly take place when the moon is rising in the constellation Aries.

139. HARVEST MOON.—The variation of the retardation of rising, according as the moon is in, or near, different parts of the ecliptic, being understood, the explanation of the *harvest moon* is very easy.

It is obvious that at all times when the moon is full she will rise nearly when the sun sets, but at the full moon nearest the autumnal equinox, the moon is observed to rise nearly at sunset, *for several nights together*. This moon, for its uses in lengthening the day, at a time when a continuance of light is most desirable to assist the husbandman in securing the fruits of his agricultural labours, is called the *harvest moon*.

140. EXPLANATION OF THE HARVEST MOON.—The moon, at full, being near the part of the ecliptic, opposite to the sun, and at the autumnal equinox the sun being in Libra, consequently the moon must be then near Aries, when, from what has been stated, the retardation of her rising on successive nights, only amounts to 14½ minutes in the latitude 53° 25′; and as the moon at full always rises at sunset, the cause of the whole phenomenon is apparent. In places near the Arctic circle, where the colatitude is 23° 28′, and where the ecliptic

consequently coincides with the horizon when Aries is rising, the phenomenon is still more striking, and there it is of greater use, where the changes of the seasons are much more rapid.[a]

141. On the Atmospheres of the Planets and Moon.—In tracing analogies between the planet on which we live and the other planets, we naturally inquire respecting their atmospheres. The atmosphere

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[a] To calculate the retardation in these cases, by spherical trigonometry; if ω be the obliquity of the ecliptic, and m the moon's mean motion in longitude (CL or CL′, Fig. 20),

sin LN. (moon's declination) = sin ω × sin m.
tan CN. (moon's R.A.) = cos ω × tan m.
sin CM. = tan lat. × tan HM. (moon's declin.)

Now CL in its mean quantity is about 12°, and therefore for lat. 53°, 23′ we shall find by actual computation,

$$\left.\begin{array}{l} CN = 11^{\circ}\ 2' \\ CM = \ 6\ \ 24 \end{array}\right\} \begin{array}{l} \text{Hence } CN - CM = 4^{\circ}\ 38' \text{ or in time} = 18^{m}\ 32^{s} \\ \text{and } CN + CM = 17^{\circ}\ 26' \text{ or in time } 1^{h}\ 9^{m}\ 44^{s}. \end{array}$$

Hence the interval between the rising of the moon on two different nights, when in Aries = $23^{h}\ 56^{m} + 18\frac{1}{2}^{m} = 24^{h}\ 14\frac{1}{2}^{m}$ nearly, and the retardation is only $14\frac{1}{2}^{m}$ mean solar time. When the moon rises in Libra, the interval is $23^{h}\ 56^{m} + 1^{h}\ 5\frac{3}{4}^{m} = 25^{h}\ 5\frac{3}{4}^{m}$, and the retardation is $1^{h}\ 5\frac{3}{4}^{m}$.*

This difference is still greater, the nearer we approach the Arctic circle, and there the retardation of rising, when in Aries, becomes smaller, for then HCN (the compl. of lat.) approaches to equality with LCN, the obliquity of the ecliptic; and therefore the points H and L approach each other, and consequently MN becomes smaller. At the Arctic circle itself, the ecliptic coincides with the horizon, when Aries is rising, and MN vanishes, and therefore the interval between two successive risings is only $23^{h}\ 56^{m}$. So that there the moon actually rises four minutes sooner than she did the previous night as measured by mean solar time.

* In the preceding demonstration, it has been supposed that the moon moves in the ecliptic, instead of a great circle inclined to the ecliptic at an angle of 5°, and that her motion in her orbit in 23^{h} 56^{m} is uniform through the month. As neither of these suppositions is strictly true, the foregoing calculation is only approximate. This effect will be increased from the inclination of the moon's orbit to the ecliptic, when the ascending node is between Capricorn and Cancer, and decreased when between Cancer and Capricorn, as in the former case the inclination of the moon's motion to the equator is increased, in the latter diminished.

which surrounds the earth has such various and important uses, that we can hardly suppose the planets destitute of an element, of which we know not whether the simplicity of construction, or the complicated advantages of it, are most to be admired.

We can ascertain that Venus, Mars, and Jupiter, are surrounded by transparent fluids, which reflect and transmit light, and are therefore, according to much probability, of the same nature as our atmosphere.

The spots and belts of Jupiter are not exactly stationary on his disc, but are observed to undergo changes and small motions similar to what would be observed, from a distance, of the clouds of our atmosphere; whence they are supposed to be clouds in his atmosphere; from some cause unknown to us, more permanent than any of the clouds of the earth. From observing the revolutions of some spots at different times, Sir W. Herschel has discovered a difference very similar to what would arise, did monsoons take place in the atmosphere of Jupiter, as they do in that of the earth. It is indeed probable that its density is such that we observe in his case the phenomena of his atmosphere alone, without ever perceiving the surface of his actual body.

But the existence of an atmosphere about Venus, as dense, or probably denser than that of the earth, seems to be put beyond all doubt, by the observations of M. Schroeter. He, for a series of years, observed Venus with great attention with reflecting telescopes of his own and of Sir W. Herschel's making, and also with achromatic telescopes.

The results of his observations are, that Venus revolves on an axis, in $23^h\ 21^m$; has mountains like the earth, and enjoys a twilight. In several favourable circumstances, when Venus was seen a thin crescent, he measured the extension of light beyond the semicircle of the crescent, and found it to be such, that the observed zone of Venus, illuminated by twilight, must have been at least four degrees in breadth. Now for the twilight to be seen by us through the atmosphere of Venus, and our own, extending through such

an arc, makes it very probable that the inhabitants of Venus enjoy a longer twilight than those of the earth, and that her atmosphere is denser. Mars has also an atmosphere, which however allows us to see the formation of the continents very plainly. Near the poles we observe white spots, which increase and diminish according to the seasons, being the largest whenever it is winter at the pole, and which must probably be attributed to masses of snow. A small star, hid by Mars, was observed to become very faint before its appulse to the body of the planet.

142. The existence of an atmosphere in some of the planets being ascertained, we are led to make inquiry with respect to the satellites. We can have little hopes of being able to ascertain the point, except in our own satellite, the moon.

Many astronomers formerly denied the existence of an atmosphere at the moon; principally, from observing no variation of appearance on the surface, like what would take place did clouds exist as with us: and also, from observing no change in the light of the fixed stars on the approach of the dark edge of the moon. The circumstance of there being no clouds, proves either that there is no atmosphere similar to that of our earth, or that there are no waters on its surface to be converted into vapour: and that of the lustre of the stars not being changed, proves that there can be no dense atmosphere.

Had the moon an atmosphere of considerable density, it would readily be discovered by the durations of the occultations of the fixed stars. The duration of an occultation would be sensibly less than it ought to be, according to the diameter of the moon. The light of the star passing by the moon would be refracted by the lunar atmosphere, and the star rendered visible when actually behind the moon; in the same manner as the refraction by the earth's atmosphere enables us to see the celestial objects for some minutes after they have actually sunk below our horizon, or before they have risen above it. Now, the duration

is certainly never lessened four seconds of time, in which time the moon moves over 2″ of space, 1″ is to be attributed to the beginning, and 1″ to the end of the occultation, which proves that the horizontal refraction at the moon must be less than 0″.5, which therefore shows that if a lunar atmosphere exists, it must be 4000 times rarer than the atmosphere at the surface of the earth, because the double horizontal refraction by the earth's atmosphere is nearly 4000″.

The existence of a solar atmosphere of considerable absorbing powers, external to the luminous surface, is plainly proved by the fact, that the borders of the sun's disc are always considerably less luminous than the central region, and by the phenomena observed in total eclipses of the sun, when it is seen to be surrounded by a bright ring of light called the corona.

143. OF THE RINGS OF SATURN.—Soon after the invention of telescopes, a remarkable appearance was observed about Saturn. After a considerable interval of time, Huygens having much improved them, discovered, by careful observations, a phenomenon unique, as far as we know, in the solar system. He found that Saturn is encompassed with a broad thin ring, inclined by a constant angle of about 28° to the plane of Saturn's orbit; and therefore at nearly the same angle to our ecliptic, and so always appearing to us obliquely. It is invisible; 1°, when its edge is turned towards us, on account of its thinness not reflecting light enough to be visible, except in the very best telescopes ; 2°, when the plane of the ring passes between the earth and sun, because its enlightened part is turned from us; and 3°, when it passes through the sun, the edge being only illuminated.

The ring is a very beautiful object, seen in a good telescope, when in its most open state. It then appears elliptical, its breadth being about half its length. Through the space between the rings and the body, fixed stars have sometimes been seen. The surface of the ring appears more brilliant than that of Saturn himself.

Fig 21 represents the orbit of Saturn round the sun,

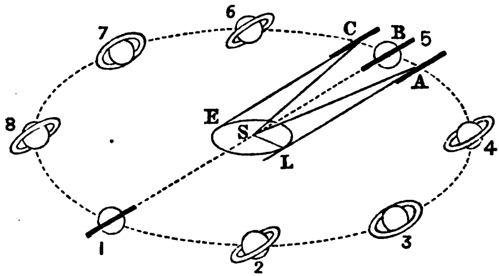

Fig. 21.

the planet being placed in eight positions of its orbit, the first position corresponding to its place in 1862, when the plane of the ring passed through the sun; the third in 1869, when the ring was most open; the fifth in 1876, when its plane will again pass through the sun; the seventh in 1883, when the ring again becomes most open.

144. It may be shown that we will usually have two appearances, and two disappearances of the rings in the course of the year, in which the plane of Saturn's ring crosses the earth's orbit in the following manner: Draw two lines LA, EC (Fig. 21), tangent to the earth's orbit, and parallel to the plane of the ring, cutting Saturn's orbit in A and C. It is evident that the angle ASC is double the angle ASB or SAL, whose sine is SL divided by SA, or 10 divided by 95 (Art. 3), that is an angle of 6° 1′; ASC is consequently 12° 2′, and as Saturn takes $29\frac{1}{2}$ years to complete his whole orbit, he will move through this angle in about 360 days, or very nearly one of our years. Consequently it follows that the plane of the ring can never pass through the earth except when Saturn is between the points of intersection of LA and EC, with its orbit, and that during that time it must do so twice.

145. Sir W. Herschel discovered that the ring which heretofore had generally been supposed single, consists

of two rings in the same place, separated by a small interval, which appears as a narrow black line. There may be even more than two rings, as astronomers have observed occasionally several such black lines parallel to the more conspicuous black divisions separating the two rings; and as these lines have been noticed on both sides of the ring, there can be very little doubt that they are lines of real separation. The thickness of the ring is very small compared with its width, and does not exceed 250 miles. The outside diameter of the exterior ring is nearly 180,000 miles, and that of the inner ring 150,000 miles, the interval between the rings is about 1800 miles. The interval between Saturn and the inner ring is about 19,000 miles.

In the year 1850 Mr. Bond, of Cambridge, U. S. A., and Mr. Dawes, of England, independently of each other, discovered a dusky or nebulous ring inside the inner of the former rings, and estimated at one-fifth of their united breadth, and within 8000 miles of Saturn. At the mean distance of Saturn, the apparent diameter of the larger ring is $47''\frac{1}{2}$.

146. ON COMETS.—Comets are luminous bodies, occasionally appearing, and generally in the part of the heavens not far from the sun. They are not as bright as the planets, but have somewhat of a nebulous appearance. They do not appear long at a time; some are seen only for a few days, and those that appear longest, only for a few months. Their often sudden appearance is, however, only caused by the position of the orbit with regard to the horizon in connexion with their rapid movement. Thus the comet of 1861 appeared very suddenly in Europe, but was visible during several weeks before in the southern hemisphere. They move about the sun in very eccentric ellipses, the sun being in one of the foci, according to the same laws as the planets; but they differ from the planets in the direction of their motion about the sun, some being direct, and others retrograde. Their paths with respect to the ecliptic are also very different: some move in a direction nearly perpendicular to it.

147. But the most striking phenomenon, and what makes them objects of attention to all mankind, is the tail of light which they often exhibit. In general when a comet appears first, it resembles a round or oval nebula. When it gradually approaches the sun it developes in its interior a portion more brilliant than the rest, which is called the nucleus. The large comets, when they approach the sun, then show luminous jets emanating from this nucleus in the direction towards the sun. These are more or less irregular, and sometimes appear all at the same time, sometimes one after the other. But they all come back after a while in the direction nearly opposite the sun, as if acted upon by a repelling force directed from the sun, and form a nebulous appendage, called the tail, which often attains an immense apparent length, as for instance that of the comet of 1618, which exceeded 100°. The direction of the tails is always opposite to the sun, but they are usually somewhat curved, and in general bend towards the region from which the comet is moving. The maximum of the splendour occurs always some days after the nearest approach to the sun. After that the comet becomes less bright, the jets of light disappear, the tail gradually vanishes, and with the increasing distance from the sun the comet assumes again the form of an ill-defined nebulous mass which it presented at its first appearance. The density of the comets is very small, as one can see even faint stars through the tail, as well as through the nebulous matter surrounding the nucleus. Even the nucleus itself presents no solid appearance, as none of them have ever presented phases, and must, therefore, be masses of vapour through which the solar light is able to penetrate. It is probable, however, that these nuclei are self-luminous, as observations have shown that the light of the comet surrounding the nucleus is alone polarized.

148. Comets are divided in two classes according to the extent of their orbits. Most of them describe ellipses or hyperbolas of such great eccentricity that the small arc of the orbit in which they can be seen does not essentially differ from the arc of a parabola,

and therefore their orbits are usually only calculated as parabolic. The number of comets whose orbits have thus been computed amounts already to several hundred. The most interesting of the comets are those which have been ascertained to move in elliptic orbits and perform their revolutions within the limits of the solar system. The number of these, however, is small. We shall defer the considerations of their orbits and motions till after we have given an account of the discoveries of Kepler; and, although the laws of planetary motion which he first brought to light might by analogy have led to a knowledge of the motion of comets, yet nothing of consequence was done till Newton himself illustrated the subject.

149. Of great interest is the discovery lately made of the connexion between comets and the meteors (shooting stars). Our knowledge about the nature of shooting stars we owe specially to Bengenberg and Brundes, who in the beginning of this century tried to find out their paths relative to the earth by noting from two stations at the extremities of a measured base line, the instants and apparent places of their appearance and extinction. They found that their height at these times varies from 16 to 160 miles, and their relative velocities from 18 to 36 miles. This great velocity led astronomers to the conclusion that they are small planetary bodies which, encountering the earth in their course, become heated while passing through our atmosphere, by the resistance of the air, and appear thus in an ignited state during that time. Although numbers of such meteors are observed every night, there are certain occasions on which they occur more frequently, and sometimes in extraordinarily large numbers. Such especially are the nights of the 14th of November, and of the 10th and 11th of August. On these nights all the meteors have the same radiant point; which for the August meteors is the constellation of Perseus, and for the November meteors a point in the constellation of Leo. The meteors do not really all start from that point, but their apparent orbits, when produced, all meet there,

excepting always a few which do not belong to these streams and are called sporadic. The earth thus encounters on these days a stream of such little bodies moving together through space in parallel orbits, the divergence of their apparent paths being a mere effect of perspective. The displays of the November meteors having been known for a long time, records of them existing as far back as the beginning of the tenth century, it was found by Professor Newton, of Yale College, that these great displays recur in cycles of 33·25 years, and when their orbit was computed by Adams from their velocity, their radiant point, and the place of the earth, it was found, that this agrees very closely with that of Temple's comet of 1866, for which also a period of 33 years had been found.[a] The orbit of the August meteors, supposing it to be a parabola or a very elongated ellipse, had likewise been found by Schiaparelli, in 1866, to resemble very closely that of the great comet of 1862, for which a period of about 130 years had been found from the observations. These coincidences, which it is impossible to consider as accidental, show that these comets form part of these remarkable streams of meteors, and open a wide field for speculation regarding their nature. It is now considered to be established that Temple's comet consists of an elliptic train or belt of minute planets revolving round the sun all in nearly the same orbit; that this

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[a] This was confirmed by the independent investigations of Schiaparelli, Peters, and Leverrier. Professor Newton had established that there is a denser part of the group of meteors, which extends over a portion of the orbit so great as to occupy $\frac{1}{10}$ or $\frac{1}{15}$ of the periodic time in passing any particular point; and he gave a choice of five different periods for the revolution of this meteoric stream round the sun, any one of which would satisfy his observed facts. He further concluded that the point in which the meteoric belt cuts the earth's orbit has a progressive motion of 52″.4 per annum. Professor Adams found by mathematical investigations that only one of these five periods permitted this progressive motion to be explained by the disturbances of Jupiter, Saturn, and other planets, and that this was $33\frac{1}{4}$ years. The greatest distance of this elliptic belt from the sun is 1798 millions of miles, and the least 87 millions.

elliptic belt contains some portions in which these little bodies are more closely packed together; that the earth's orbit intersects this belt on the 14th of each November, and that thirteen times from Oct. A.D. 902, to Nov. 14, 1866, we have gone through a denser part of the train.

150. The appearance of one comet has been several times recorded in history, viz., the comet of 1680. The period of this comet is 575 years. It exhibited at Paris a tail 62° long, and at Constantinople one of 90°. When nearest the sun it was only $\frac{1}{6}$ part of the diameter of the sun distant from his surface; when farthest, its distance exceeded 138 times the distance of the sun from the earth.

151. When the theory of the motion of comets was understood, Dr. Halley examined the comets that had been previously recorded in history, and been observed by astronomers. In general, he found the circumstances so vaguely delivered, or the observations so inaccurately made, that he was able to determine with much probability the identity of only one. The comet which Dr. Halley predicted with a degree of confidence, returned in 1759. It had been previously observed with accuracy, in 1682 and 1607, and had also been noticed in 1531, 1456, and 1305. Its return was anxiously looked for by astronomers, and some curious circumstances attending it will be afterwards noticed. With what satisfaction it was received by the scientific part of mankind may easily be conceived, and how strikingly contrasted with the reception of the same comet in 1456, when all Europe beheld it with fear and amazement. The Turks were then engaged in the successful war in which they destroyed the Greek empire; and Christians in general thought their destruction portended by its appearance. It was expected that this comet would re-appear in 1835, and astronomers calculated beforehand that the time of its approaching nearest to the sun would be the 14th November of that year. Its perihelion passage was actually the 16th of November.

CHAPTER IX.

ON ECLIPSES OF THE SUN AND MOON—TRANSITS OF VENUS AND MERCURY OVER THE SUN'S DISC.

152. THE eclipses of the sun and moon, of all the celestial phenomena, have most and longest engaged the attention of mankind. They are now in every respect less interesting than formerly; at first they were objects of superstition; next, before the improvements in instruments, they served for perfecting astronomical tables; and last of all, they assisted geography and navigation. Eclipses of the sun attract now considerable attention from the striking phenomena which the sun presents in a total eclipse, and which have enabled us to ascertain the nature of its luminous atmosphere.

153. ECLIPSES OF THE MOON.—An eclipse of the moon being caused by the passage of the moon, when in opposition to the sun, through the conical shadow of the earth, the magnitude and duration of the eclipse depend upon the length of the moon's path in the shadow.

Let AB and TE (Fig. 22) be sections of the sun

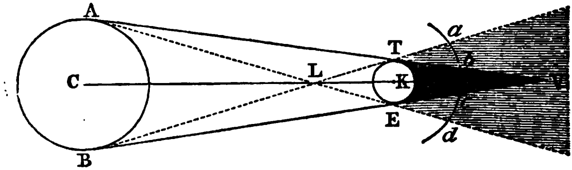

Fig. 22.

and earth by a plane, perpendicular to the plane of the

ecliptic. Let ATV and BEV touch these sections externally, and BL*a* and AL*d* internally. Let these lines be conceived to revolve about the axis CKV; then TVE will form the conical shadow, from every point of which the light of the sun will be excluded. The spaces between T*a* and TV, and between VE and E*d* will form the *penumbra*, from which the light of part of the sun will be excluded, more of it from the parts near TV and EV than those near T*a* and E*d*.

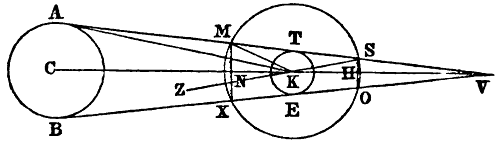

Fig. 23.

The semi-angle of the cone (TVK) = sem. diam. sun (CKA) − horizontal parallax of the sun (TAK). The angle subtended at the earth by the semi-diameter of the section of the shadow SH at the moon's distance KS = SKH = ZKC = ZKA − AKC = (KST + KAT) − AKC = horizontal parallax of the moon + horizontal parallax of the sun − semi-diameter of the sun.

The angle of the cone being known, the height of the shadow may be computed. For KT = KV. sin KVT = KV sin AKC nearly, since the sun's horizontal parallax is only 8″.6; also the diameter of section of the shadow at the moon is known, for SH or $\frac{1}{2}$ SO = SK × sin SKH.

The height of the shadow varies from 213 to 220 semi-diameters of the earth, and nearly varies inversely as the apparent diameter of the sun. The average length is 856,200 miles.

154. When the moon is entirely immersed in the shadow, the eclipse is total; when only part of it is involved, partial; and when it passes through the axis of the shadow, it is said to be central and total. The breadth of the section of the shadow at the distance of the moon is about three diameters of the moon; there-

fore when the moon passes through the axis of the shadow, it may be entirely in the shadow for nearly two hours, since it moves through a space equal to its own diameter in about an hour (Art. 125).

The angle SKH is, when greatest, about 45′, since in order that it should be the greatest possible, the moon's horizontal parallax should be the greatest, and the sun's apparent semi-diameter least, or the moon should be at the least, and the sun the greatest distance from the earth at the same time. In this case the moon's parallax is 60′ 13″. The sun's parallax 8″, and the moon's semi-diameter 15′ 45″, therefore SHK = 44′ 36″: consequently, as the moon's latitude is sometimes above 5°, it is evident an eclipse of the moon can only take place when it is near its nodes.

155. Calculation of Circumstances of a Lunar Eclipse.—The circumstances of an eclipse of the moon can be readily computed. The latitude of the moon at opposition, the time of opposition, the horizontal parallax of the moon, and diameters of the sun and moon are known at all times from the astronomical solar and lunar tables. Let the circle OCK (Fig. 24) represent the section of the shadow at the moon, HN the path of the centre of the moon, NOC the ecliptic, and CL the latitude of the moon at opposition. While the moon moves from L to N in her orbit, the centre of

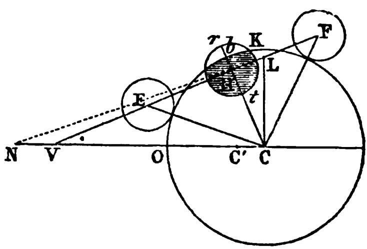

Fig. 24.

the shadow will move from C to C′, in consequence of the sun's motion, and the angular velocity of the moon being about $13\frac{1}{3}$ times the apparent angular velocity of the sun (or of the centre of the shadow), they will approach to each other more than if C were at rest. In order to calculate this *relative* motion

let us reduce C to rest by applying to both C and the moon a motion equal to that of C, and parallel to the ecliptic in the opposite direction, or C′C. If we take HN to NV as the angular velocity of the moon to that of the sun, and join HV, it is evident that HV will represent the path of the moon relative to the centre of the shadow. The eclipse will begin at F, and end at E, when CF or CE, the distance of the centre of the moon from the centre of the shadow, will be equal to the semi-diameter of the moon, added to that of the section of the shadow, or the semi-diameter of the moon + horizontal parallax of the moon + horizontal parallax of the sun − sun's semi-diameter (Art. 153), and the eclipse is at its greatest when the distance between the centres is the least possible; that is, when the centre of the moon is at H, the foot of the perpendicular CH. In the right-angled triangle CHL we know CL and HCL (= HVC the inclination of the lunar orbit nearly). Hence we find HC and HL. HC never differs more than a few seconds from CL. From HC and CF (the sum of the semi-diameters of the section of the shadow and moon) we compute FH (= HE, and thence EL (= EH + HL) and LF (HF − HL). Thence knowing the time when the moon is in opposition at L, the spaces FL and LE, and the *relative* velocity of the moon in the path HV, we can find the time of describing FL and LE, or the time from the beginning of the eclipse to opposition, and the time from opposition to the end; and so the times of beginning and ending of the eclipse are known from the time of the moon's opposition.

If the diameter *rb*H*t* of the moon be divided into twelve equal parts, called digits; then, according to the number of these in *bt*, the eclipse is said to be of so many digits.

156. TO FIND THE LUNAR ECLIPTIC LIMITS.—The greatest distance of the moon, at opposition, from its node, that an eclipse can happen, is above $11\frac{1}{2}°$, and is called its ecliptic limit. When the moon is nearest the earth, and therefore circumstances are *most favour-*

able for an eclipse, let EA (Fig. 25) represent the semi-

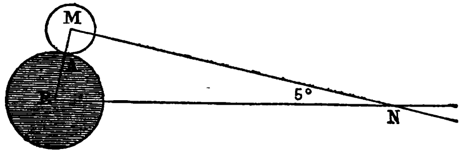

Fig. 25.

diameter of the shadow at the moon, and MA the semi-diameter of the moon touching it; MN the apparent path of the moon, and N the place of the node. Then EN is the extreme limit of the distance of the node from conjunction, at which the eclipse can happen. In the spherical triangle MEN we have ME (when greatest) 1°2′38″, and the angle N (when least) 4° 57′ 22″, and the angle at M right, consequently EN = 12°4′. If the moon be farthest from the earth, and the angle at N the greatest, and consequently circumstances be *least favourable* for an eclipse of the moon, ME = 52′20″; N = 5° 20′ 6″ and EN (the minor ecliptic limit) = 9°30′. If the distance of the centre of the shadow from the node be greater than 12°4′ an eclipse cannot happen; if less than 9°30′ it must occur.

157. If the moon's nodes were fixed, eclipses would always happen at the same time of the year as we find the transits of Venus and Mercury do, and will continue to do for many ages: but as the nodes perform a revolution backward in about 18½ years, the eclipses happen sooner every year by about nineteen days.

Chaldean Saros.—In 223 lunations, or eighteen years, 10 days, 7 hours, and 43 minutes, or 18 years, 11 days, 7 hours, and 43 minutes, according as there are five or four leap years in the interim, the moon returns to the same position nearly with respect to the sun, lunar nodes, and apogee, and therefore the eclipses return nearly in the same circumstances: this period was called the Chaldean Saros, being used by the Chaldeans for foretelling eclipses. If therefore we record the eclipses of the moon in order for a period of 18 years and 11 days, we can approximately find those of

the next similar period, as they will recur in the same order.

158. From the refraction of the sun's light by the atmosphere of the earth, we are enabled to see the moon in a total eclipse, when it generally appears of a dusky red colour. The rays of light which fall from the sun on the upper part of the atmosphere are bent by it, and entering the shadow fall upon the moon. The moon has, it is said, entirely disappeared in some eclipses.

The gradual diminution of the moon's light as it enters the Penumbra makes it very difficult to observe accurately the commencement of a total eclipse of the moon. It is also necessary to increase the diameter of the shadow, in consequence of the earth's atmosphere stopping the solar rays, by about $1'30''$. An error of above a minute of time may easily occur. Hence lunar eclipses now are of little value for finding geographical longitudes. The best method of observing an eclipse of the moon is by noting the time of the entrance of the different spots into the shadow, which may be considered as so many different observations.

159. Eclipses of the Sun.—From what has been said of the earth's shadow, it is easy to see that the angle of the moon's shadow, when it comes between the earth and the sun, is nearly equal to the apparent diameter of the sun. We may compute the length of the conical shadow of the moon on the same principles as in the case of that of the earth, and we find that it varies from $60\frac{1}{2}$ to $55\frac{1}{2}$ semi-diameters of the earth. The moon's distance varies from 65 semi-diameters to 56. Thus when the moon is nearest to the earth the vertex of the shadow may lie beyond the earth's centre by a space equal to $3\frac{1}{2}$ times the earth's radius. Therefore sometimes when the moon is in conjunction with the sun, and near her node, the shadow of the moon reaches the earth, and involves a small portion in total darkness, and so occasions to the inhabitants of that part a total eclipse of the sun. The part of the earth involved in total darkness is always very small, it being so near the vertex of the cone; but the part involved in the Penumbra extends over a considerable portion of the hemisphere turned

towards the sun: in these parts the sun appears partially eclipsed.

160. The length of the shadow being sometimes less than the moon's distance from the earth no part of the earth will be involved in total darkness; but the inhabitants of those places near the axis of the cone will see an annular eclipse—that is, an annulus of the sun's disc will only be visible. Thus let HF, LU (Fig. 26) be sec-

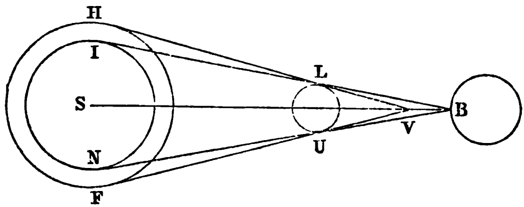

Fig. 26.

tions of the sun and moon. Produce the axis SV of the cone, to meet the earth in B: from B draw tangents to the moon, intersecting the sun in I and N. The circle, of which IN is the diameter, will be invisible at B, and the annulus, of which IH or NF is the breadth, will be visible.

It has been computed[a] that a total eclipse of the sun can never last longer, at the earth's equator, than $7^m\ 58^s$, nor be annular longer than $12^m\ 24^s$. The diameter of the greatest section of the shadow that can reach the earth is about 166 miles.

161. The general circumstance of a solar eclipse may be represented by a projection with considerable accuracy, and a map of its progress on the surface of the earth constructed.

The phenomena of a solar eclipse at a given place

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[a] Thus: the greatest value of the moon's semi-diameter is 1025″ and the least value of that of the sun's 945″. The difference is 80″. The greatest duration of total eclipse would be the time which the moon would take to describe twice this difference or 160″. Since its greatest motion is not quite 34″ in a minute (Art. 125) this would take not quite five minutes; but the earth's rotation in the same direction increases it as above.

may be well understood by considering the apparent diameters of the sun and moon on the concave surface, and their distances as affected by parallax. When the apparent diameter of the sun is greater than that of the moon, the eclipse cannot be total, but it may be annular.

From the solar and lunar tables we compute for the given place the time when the sun and moon are in conjunction—that is, have the same longitude. From the horizontal parallax of the moon, given by the tables, at this time, we compute its effects in latitude and longitude; by applying these to the latitude and longitude of the moon, computed from the tables, we get the apparent latitude and longitude, as seen on the concave surface; and knowing the longitude of the sun, we compute the apparent distance of their centres, from whence we can nearly conclude the time of the beginning and ending of the eclipse, especially if we compute by the tables the apparent horary motion of the moon in latitude and longitude at the time of the conjunction. About the conjectured time of beginning compute two or three apparent longitudes and latitudes, and from thence the apparent distances of the centres, from which the time may be computed by proportion when the apparent distance of the centres is equal to the sum of the apparent semi-diameters—that is, the beginning of the eclipse. In like manner the end may be determined. The magnitude also of the eclipse at any time may be thus determined: let SE (Fig. 26) be the computed apparent difference of longitude of the centres L and S, LE the computed apparent latitude of the moon. In the triangle LSE we have therefore LE and ES to find SL the distance of the centres. Hence mn (the breadth of the eclipsed part of the sun) = $Ln + Sm - SL$ is known.

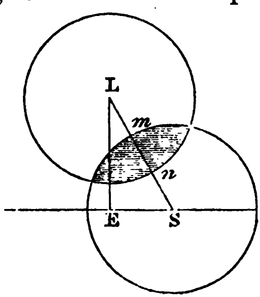

Fig. 27.

162. Solar Ecliptic Limits.—The ecliptic limits of

the sun (the greatest distance of the conjunction from the node of the moon's orbit, when an eclipse of the sun can take place) may be found as follows:—(See Fig. 23, p. 117). Let the moon's orbit intersect the common tangents to the sun and earth in M and X; when the moon is in conjunction, and just touches the cone formed by common external tangents AT, BE to these bodies, there will be no eclipse; now the angle MKN = MKA + AKC = (TMK − MAK) + AKC = hor. parall. moon − hor. parall. sun + sun's semi-diameter; when the distance of the centres of the sun and moon is greater than this angle + the moon's semi-diameter there will be no eclipse. Proceeding as before we get the greatest value of this distance 1° 35′14″, and the least 1° 24′19″, and the corresponding ecliptic limits, 15°25″ distance of the moon from the node within which an eclipse must take place, and 18° 20′ distance beyond which it cannot possibly occur.

163. There must be two eclipses, at least, of the sun every year, because the sun describing on the average in a month 30° 37′ in reference to the moon's node, is above a month in moving through the solar ecliptic limits 30° 50′. But there may be no eclipse of the moon in the course of a year, because the sun is not a month in moving through the lunar ecliptic limits (24° 8′ at the greatest).

When a total and central eclipse of the moon happens, there may be solar eclipses at the new moon preceding and following, because, between new and full moon, the sun moves in reference to the node only about 15° 18′, and therefore the preceding and following conjunctions will be at less distances from the node than the limit for eclipses of the sun 15° 25′. As the same may take place at the opposite node, there may be six solar eclipses in a year. Also when the first eclipse happens early in January, another eclipse of the sun may take place near the end of the year, as the nodes retrograde nearly 20° in a year. Hence there may be seven eclipses in one year, five of the sun, and two of the moon, as in the year 1823. In 18 years there are 41 eclipses of the sun, and 29 of the moon generally.

256. As a solar eclipse will take when the distance between the centres of the moon and sun is less than MKN + the moon's semi-diameter, and a lunar eclipse when the distance of the centres of the moon and centre of the section of the shadow is less than SKH + moon's semi-diameter; and since MKN exceeds SKH by twice the difference of the sun's semi-diameter and parallax, an eclipse of the sun will be more likely to occur than one of the moon; but the latter, when it does occur, is visible over a hemisphere, the former over a limited portion of the earth.

A total eclipse of the sun, April 22, 1715, was seen in most parts of the south of England. A total eclipse of the sun had not been seen in London since the year 1140.

The eclipse of 1715 was a very remarkable one; during the total darkness, which lasted in London $3^m\ 23^s$, the planets Jupiter, Mercury, and Venus were seen; also the fixed stars Capella and Aldebaran. Dr. Halley has given a very interesting account of this eclipse,[a] which is said by Maclaurin to be the best description of an eclipse that astronomical history affords. A particular account is also given in the Phil. Trans. by Maclaurin of an annular eclipse of the sun, observed in Scotland, Feb. 18, 1737. He remarks, that this phenomenon is so rare, that he could not meet with any particular description of an annular eclipse recorded. This eclipse was annular at Edinburgh during $5^m\ 48^s$. The next total eclipse visible in England will be on August 11th, 1999.

257. The beginning and end of a solar eclipse can be observed with considerable exactness, and are of great use in determining the longitudes of places; but the computation is complex and tedious, from the necessary allowances to be made for parallax.

258. Transits of Venus and Mercury.—The planets Venus and Mercury moving round the Sun, in orbits which are slighly inclined to that of the Earth, are sometimes in inferior conjunction when near their nodes, or the intersections of their orbits with the plane of the

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[a] Phil. Trans., vol. 29.

ecliptic: they then pass over the Sun's disc, and appear as dark and well-defined spots on his body. Mercury can only be seen by the assistance of a telescope; but Venus may be seen by the eye, defended with a smoked glass, or on the image of the sun formed in a dark room by an aperture in the window. Venus appears in a telescope, a well-defined black spot, 57″ in diameter. The diameter of Mercury is only about 11″.

259. The transits of Mercury are much more frequent than those of Venus. This is merely accidental; arising from the proportion of the periodic time of Mercury to that of the Earth, being nearly expressed by several pairs of small whole numbers. If an inferior planet be observed in conjunction near its node (or in a certain place of the ecliptic), it will be in conjunction in the same place of the ecliptic, after the planet and the earth have each completed a certain number of revolutions. Now it is easily computed from the periodic times of Mercury and the Earth, that nearly

7	per.	of the Earth's rev.	=	29	per.	of Mercury's.
13	,,	of the Earth	=	54	,,	of Mercury.
33	,,	of the Earth	=	137	,,	of Mercury.

Therefore transits of Mercury, at the same node, may happen at intervals of 7, 13, 33, &c. years.

8 per. of the Earth's rev. = nearly 13 per. of the rev. of Venus for 365·256 days × 8 = 2922·05 and 224·701 days × 13 = 2921·11.

There are no intervening whole numbers till

235 per. of the earth = nearly 382 per. of Venus, for each equals 85835½ days.

Hence a transit of Venus, at the same node, may happen after an interval of 8 years. If it does not happen after an interval of 8 years, it cannot happen till after 235 years.

At present the ascending node of Venus, as seen from the sun, is in 75°, and the descending node in 255°. The earth, as seen from the sun, is in the former longitude on the 7th of December, and in the latter on the 5th of June. Hence the transits of Venus will

happen for many ages to come in December and June. Those of Mercury will happen in May and November.

161. A transit of Mercury happened at the descending node in May, 1832, and the next at that node in 1845. One happened in 1815 at the ascending node, another in 1822, and a transit at that node in 1835.

In the years 1761 and 1769 there were transits of Venus, Venus being in her descending node: the next transit at that node will happen in the year 2004. But a transit was observed at the ascending node in the year 1639 by Horrox, who had previously computed it, from having corrected the tables of Venus by his own observations, all other astronomers having been ignorant of its occurring. This transit will again happen at the end of 235 years from that time, or in the year 1874.

168. Horrox, who resided near Liverpool, when quite a youth, engaged in the study of astronomy with extraordinary enthusiasm and success. His having improved the tables of the motion of Venus so as to predict and observe this curious phenomenon, is one of the least of his astronomical performances.

He wrote an account of his observation in a dissertation, entitled, "Venus in sole visa," which, many years after his death, was published by Hevelius at Dantzic. This roused the attention of his countrymen to make inquiries respecting him, and to examine whether any of his manuscripts were remaining. A small part only of what were known to have existed were found, and were published by Dr. Wallis about 30 years after his death.[a] Thus had not his manuscript, "Venus in sole visa," accidentally fallen into the hands of Hevelius, there

[a] The account Dr. Wallis has given of the fate of Horrox's manuscripts is interesting. Some were brought to Ireland by his brother, who died here; these have never been found. Many were burned, during the civil wars of England, by some soldiers, who, searching for plunder, found them where they had been concealed. Some were used in composing a set of astronomical tables, called the *British Tables*, published in 1653. These were afterwards destroyed in the great fire at London, in 1666. The part that Dr. Wallis has published was found in the ruins of a house at Manchester, in which his friend Crabtree had resided many years before.

is reason to suppose, that in a few years scarcely any trace of this extraordinary young man would have remained. The apparent neglect of his countrymen must be attributed to the civil wars, which almost immediately followed his death. He had no assistance in his labours, except from a friend, of the name of Crabtree, who lived at the distance of 20 miles. He also cultivated, with much ardour and ability, this science. Their correspondence is extant. Crabtree, informed by Horrox, observed the transit at his own place of abode. Horrox died at the early age of 22, in the year 1641: and from what we see of his works that remain, it appears highly probable that, had his life been longer spared, his fame would have surpassed that of all his predecessors. He seems to have been the first astronomer who reduced the sun's parallax to nearly what it has since been determined. All astronomers before Kepler had made it more than two minutes: Kepler stated it at 52″: but Horrox, by a variety of ingenious arguments, evincing his superior knowledge in the science, shewed it highly improbable that it was more than 14″[a]. He also supposed that the disc of Venus, when seen on the sun, would not subtend a greater angle 1′: whereas, according to Kepler, it would be 7′. Horrox, soon after he had entered on this science, was convinced by his own observations of the value of Kepler's discoveries.

262. The transits of the inferior planets afford the best observations for obtaining accurately the places of their nodes, and also the best observations for determining their mean motions.

The transits of Venus also afford us far the most accurate method of ascertaining the sun's distance from the earth, and therefore the magnitude of the whole system.

Dr. Halley first proposed this method of finding the

[a] Dr. Halley, above sixty years after, by arguments not very dissimilar to those of Horrox, endeavoured to show that it was not more than 12½″.

sun's distance in 1716. He had observed, at the island of St. Helena, a transit of Mercury over the sun's disc, and thence had concluded that the total ingress and the beginning of the egress of Venus might be observed to 1^s of time: from whence, as he said, the sun's distance might be determined within $\frac{1}{500}$ of the whole distance. Knowing that the next transits of Venus would occur in 1761 and 1769 he urged astronomers who should be then living to make accurate observations of them for this purpose. Experience afterwards showed, that the times of total ingress and the beginning of egress could not be observed with certainty nearer than three or four seconds.

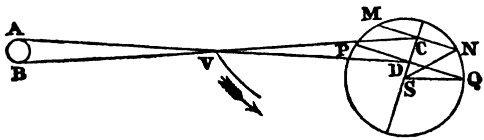

Fig. 28.

171. The exact calculations connected with the problem of calculating the sun's distance from an observation of the transit of Venus are very complicated. It is done in the following manner. Let V be Venus in inferior conjunction to the sun near the node of her orbit, and A and B two stations on the earth's surface, one as near to the north pole, and the other to the south pole as possible. As Venus is then retrograde (98), her apparent motion with respect to the sun will be in the direction PQ, or from left to right of the figure, and her apparent velocity across the sun's disc will be that with which the sun and Venus move to meet each other, and which is at that time about 4″ in a minute of time. Now, although we are not supposed to know the actual distances of Venus and the earth from the sun in miles, we know (Art. 96) the *ratio* of these distances, and consequently the *ratio* of VD to AV, this is as 723 to 277, or in the proportion of 2·61 to 1. We also know the distance between A and B in miles. Now, the observer

at A will see Venus to cross the sun's disc in the line PQ, and the observer at B will see the transit to take place in the line MN parallel to PQ; let SDC be the perpendicular to these lines from the centre of the sun —DC, in miles, will be to AB, in miles, as VD is to VA, or as 2·61 to 1. Consequently the number of miles in DC is known. The observers at A and B noting the times of the beginning and ending of the transits at the two places can ascertain the duration of the transits PQ, MN, and knowing the times of describing the spaces PQ, MN and the velocity, 4″ in a minute, we know the number of seconds of arc in these spaces; we also know the number of seconds of arc in the sun's apparent diameter, consequently the ratios of CN to SN and of DQ to SQ or the sines of the angles NSC, QSD, and consequently their cosines. Now if R be the sun's semidiameter in miles R cos NSC—R cos QSD = SC—SD = DC (which is known) consequently R (in miles) = DC divided by (cos NSC—cos QSD).

Having found the number of miles in the sun's semidiameter, and the number of seconds in the angle which it subtends at the earth, we can find the sun's distance, because the sun's semidiameter miles : sun's distance in miles : : sun's semidiameter in seconds : 206265.″ From this may be deduced the sun's parallax.

172. To explain this method in another way, let us consider Venus and the sun as moving in the equator, and that observations of the total ingress are made at two places in the terrestrial equator: let AB (Fig. 29) be the equator, S and V discs of the sun and Venus, perpendicular to, and as seen from the equator. To a spectator at A the internal contact (or the total ingress) commences, when to a spectator at B the edge of Venus is distant from the sun by the angle VBS. The difference then between the times of total ingress, as seen from B and A, is the time of describing VBS by the ap-

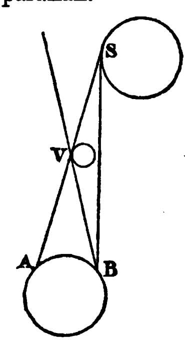

Fig. 29.

proach of the sun and Venus to each other, Venus being retrograde and the sun direct. Hence from this difference of times, and the rate at which Venus and the sun approach each other, we find VBS. And the sine of VBS : sine of VSB : : Venus's distance from the sun: Venus's distance from the earth. The relation of Venus's and the earth's distance from the sun, as found by the method in Art. 96, may be used. Therefore the angle VSB,[a] the angle subtended by the two places A and B at the sun is known, and consequently the angle the semidiameter of the earth subtends, will be found in a manner similar to that Art. 59.

173. This simplification of the problem may serve for an illustration, and to point out its superior accuracy. But the actual computation of the transit is very complex, principally on account of the inclination of Venus's orbit to the ecliptic, and on account of the situations of the places of observation at a distance from the equator. The accuracy of the method consists in this: that the times of internal contact can be observed with great exactness, and thence the angle VBS computed, and therefore ASB.

At inferior conjunction, the sun and Venus approach each other at the rate of about 240″ in an hour, or 4″ in a minute. Hence if the time of contact be erroneous at *each* place 4ˢ of time, the angle VSB *may be* erroneous $\frac{4 \times 8}{60} = \frac{8}{15}$ of a second, and therefore the limit of the error of ASB about $\frac{4}{15}$ of a second.[b]

174. This method then in fact comes to the same as to find the angle at the sun, subtended by two distant places

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[a] For extreme accuracy the distance of the places A and B is to be diminished by the arc of the equator, described in the interval of the ingresses at each place.

[b] This comes to the same, as being able to observe a thread of light (the interval between the limbs of Venus and the sun, when the former has just entered upon the body of the sun) of only $\frac{1}{4}$ of a second in breadth. Thus by the transit of Venus we can probably measure a smaller angle than by any other method.

on the earth's surface; but this angle can be determined much more accurately by the times of ingress, than by the micrometer. On account of the difference of the apparent magnitudes of Venus and Mercury, the internal contact of the former can be determined much more accurately than of the latter.

This method requires the difference of longitude of the places to be accurately known, in order to compare the actual times of contact. The longitude of the Cape of Good Hope being well ascertained, observations of the transit of Venus in 1761, made there, were compared with many made in Europe, and the mean result gave the parallax = 8.47 seconds.

175. But it seemed more convenient not to depend on the knowledge of the difference of longitudes of two places. It appeared better to compare the differences of duration at two places, at one of which the transit was lengthened and at the other shortened. If we assume the parallax of the sun, which we know nearly, we can compute the difference of duration at any place from what it would have been, had it been observed at the earth's centre.[a] Since the parallax of Venus is greater than that of the sun, the former will be more depressed than the latter, and this difference will be more sensible, the nearer they are to the horizon, when the parallax of each is the greatest; consequently, when we remember that the motion of Venus relative to that of the sun is then from east to west, it is obvious that when the transit is about to commence near sunset, Venus will be thrown down on the sun's disc, by parallax, before the transit has commenced as seen from the centre of the earth; if the transit end near sunset the planet, being now to the west of the sun, will be thrown off the sun's disc before it has really ended. The opposite effects will be produced if the transit begin or end near sunrise; the beginning or ending of the transit will be retarded by the effect of parallax.

[a] See Dr. Maskelyne's Method and Computation, page 398 of Professor Vince's Astr. vol. i.

Hence we can compare the difference of duration at two places, at one of which the duration is shortened and at the other lengthened. Thus we shall have a double effect of the parallax, and we can compare the computed result with the difference observed. From the error we can correct the horizontal parallax assumed.

The transit of Venus in 1769 took place on June 3rd, a day near the summer solstice; it was observed at Wardhus in Lapland, lat. 70° 22′ N., and at the island of Otaheite in the South Sea, lat. 17° 25′ S.

Assuming the sun's parallax 8·83 seconds,

By computation the duration was lengthened at Wardhus, . . .	$11^m\ 16^s{\cdot}9$
Diminished at Otaheite, . . .	$12^m\ 10^s{\cdot}0$
Duration greater at Wardhus than at Otaheite, ...	$23^m\ 26^s{\cdot}9$
By observation, ...	$23^m\ 10^s{\cdot}0$

This shows the parallax is less than the parallax assumed, and to make the observed and computed difference of durations agree, the parallax must be taken 8″·57. This last conclusion points out the accuracy of which the method is susceptible. The difference of excess of duration of 17^s makes only a difference of $\frac{11}{100}$ of a second in the parallax.

176. The observations of the transit of 1761 were not so well adapted for determining the sun's parallax as those of 1769. From the latter the parallax was ascertained with greater exactness. The mean of the results seems to give 8″.57 the sun's parallax at the mean distance, which probably is within $\frac{4}{10}$ of a second of the truth. The transit of 1769 occurring in the middle of summer, very many places of high northern latitude were well situate for observing it, but in all these the duration was affected in the same way.

The duration is most lengthened when the commencement is near sunset, or when the sun is near the western horizon, and the end near sunrise, or when the sun is near the eastern horizon. The duration of the transit in June, 1769, was about six hours. That the commencement and end should take place under the

circumstances above mentioned, it evidently required that the place of observation should have considerable north latitude, hence Wardhus was selected.

The duration would be most shortened when the commencement was near sunrise, and end near sunset, and the duration being only about six hours, this required that the days should be shorter than the nights, and therefore the place must be on the south side of the equator, and such that the commencement must be after sunrise, and end before sunset. Consequently the choice of situations was much circumscribed.

Astronomers were therefore much at a loss for a proper place for observing this transit, when fortunately Otaheite was discovered. In consequence of which, the first of the celebrated voyages of Cook took place. The transit commenced at Otaheite about half past nine in the morning, and ended about half past three in the afternoon, and thus it happened during the most favourable part of the day. It has since been ascertained that the observations made in Lapland cannot be depended on, and consequently that the sun's parallax and distance, as determined from the transit of 1769, are inaccurate. A parallax of $8''\cdot57$ would give the sun's distance 95,300,000 of miles. We have reason to think that the parallax is $8\cdot93''$, and his distance 93,098,796 miles.

The *ratio* of the distances of Mars and the earth from the sun is found by the method of Art. 96, to be as 3 : 2; consequently when Mars is in opposition to the sun the distance of the planet from the earth is one-half of the distance of the earth from the sun. The former of these distances is found by the method noticed in Art. 60; hence the latter is concluded to be 91,533,000 miles, which would correspond to a parallax of $8\cdot93''$ (note, page 50). Astronomers are anxiously waiting for the transit of 1874 to determine the matter accurately.

CHAPTER X.

OBSERVATIONS FOR ASCERTAINING THE DECLINATION—MURAL CIRCLE—DISTANCE OF THE POLE FROM ZENITH—OBLIQUITY OF ECLIPTIC—RIGHT ASCENSION—TRANSIT INSTRUMENT—ALTITUDE AND AZIMUTH INSTRUMENT—EQUATORIAL—OBSERVATORY CLOCKS.

177. Previously to a more minute statement of the motions of the celestial bodies, it will be necessary to give some account of the nature of the principal observations, by which these motions are ascertained, and of the instruments by which the observations are made.

The most important observations, and which admit of the greatest accuracy, are those for the declination and right ascension. Having obtained the declination and right ascension, or the position with respect to the celestial equator, we can by spherical trigonometry obtain the longitude and latitude, or the position with respect to the ecliptic (see Chap. I.) The latitude and longitude of any of the bodies of the solar system, as they would be observed from the centre of the earth, are called their *geocentric* latitude and longitude.

The astronomical tables give the distance of the body from the sun, and its place, as seen from the sun, or its *heliocentric* longitude and latitude, from whence we can compute its geocentric latitude and longitude, and compare them with those observed.

178. The *declination* of an object is best found by observing its distance, when on the meridian, from the zenith or from the horizon. This may be done by the *mural circle* (Fig. 30), which is a vertical graduated circle (see Art. 195), made of brass, attached to a horizontal

axis passing through its centre. The axis is made to point exactly east and west, so that the circle moves in the plane of the meridian. A telescope is fixed to the circle in its plane, moves with it, and passes through its centre. The axis is supported by a stone pier or wall, which is built north and south, or in the plane of the meridian; so that the plane of the circle is parallel to the wall, and nearly touches it. By means of this a star is observed only when on the meridian, and as the telescope points to the star, the graduated arc will show its angular distance from the horizon or the zenith. Either of these distances being found, if we previously know the distance of the pole from the zenith of the place, or the co-latitude, we find by addition or subtraction the distance of the object from the elevated pole, and consequently its declination. Thus it is evident that the distance of the star from the North Pole, when on the meridian, is equal to the co-latitude + the zenith distance. Similarly the co-latitude ± the declination is equal to the altitude. There is an advantage in using the polar distance instead of the declination, because in the former there is no ambiguity; but when the declination is used, it is necessary to note whether it be north or south, the sign + being used in the former case, – in the latter. Accordingly many astronomers use in their catalogues of stars north polar distances instead of declination: thus, if the declination be 20° S. its north polar distance is 110°.

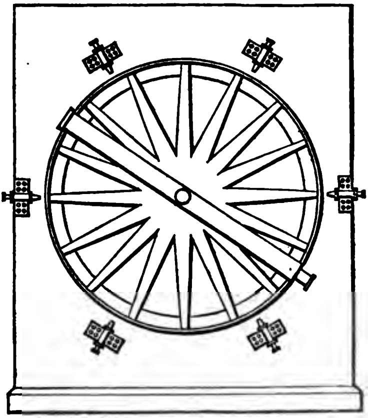

Fig. 30.

It must be understood that the zenith distance, or altitude observed, is to be corrected for refraction, and by some other small quantities, as sometimes for parallax (to reduce it to what would be observed at the earth's centre), for aberration of light and nutation of the earth's axis; which corrections are usually obtained from tables.

179. The distance of the zenith, Z, from the pole is found by observing the zenith distance of a star that does not set, when on the meridian above and below the pole: thus let Zp (Fig. 5, p. 35), be the zenith distance, corrected for refraction, of a star, when on the meridian above the pole, N; and Zp' the zenith distance corrected for refraction, when below the pole N; then $ZN = Zp + pN$ and $ZN = Zp' - p'N = Zp' - pN$. Hence $2ZN = Zp + Zp'$. NO, as we have seen (Art. 39), is equal to the latitude of the place, and therefore ZN is the complement of latitude. Hence, the observations for ascertaining the distance of the pole from the zenith give us the latitude of the place of observation.

By repeating this observation for the same star, and for different stars, a great many times, the distance of the pole from the zenith may be had with great exactness, that is, with good instruments, to a fraction of a second. This element being once established, we are enabled, as stated above, to obtain by observation the declination or polar distance of any celestial phenomenon. It is necessary in this mode of observation to know with some degree of exactness when the object is at the meridian; this will be explained hereafter.

180. In computing the longitude and latitude of an object, from knowing its right ascension and declination, we use the obliquity of the ecliptic. The obliquity of the ecliptic is found by observing the greatest declination of the sun. If many declinations be observed when the sun is near the solstice, each of these may by a small correction be reduced to the declination at the solstice, and the mean of all taken. The advantage of this is, that the declination observed

within a few days of the solstice may easily be reduced to the greatest declination, without knowing with great accuracy the right ascension of the sun. The summer solstice is to be preferred to the winter one, on account of refraction being more uncertain at lower latitudes.

181. To ascertain the right ascension of an object, it is necessary to find the arc of the equator intercepted between the first point of Aries and a secondary to it passing through the object: for this purpose we make use of a portion of duration, called sidereal time. The whole concave surface appears to revolve uniformly in twenty-four hours of sidereal time, and any portion of the equator is measured by the sidereal time elapsed between the passages of its extremities over the meridian: thus the extremities of an arch of 15° pass the meridian at an interval of one hour. Hence we conclude that the difference of right ascension of these extremities is 15° or one hour: so that the right ascension of any object is measured by the portion of sidereal time elapsed between the passages or transits of the first point of Aries (the intersection of the ecliptic and equator), and of the object over the meridian. Hence if a clock be adjusted to show twenty-four hours during the rotation of the concave surface, and commence its reckoning when the first point of Aries is on the meridian, it will show the right ascension of all the points of the concave surface on the meridian at any time; and all that is necessary to ascertain the right ascension of any object is to observe the time shown by the clock

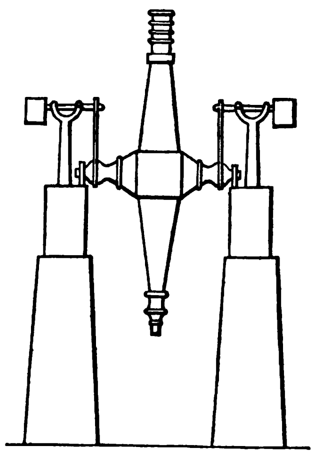

Fig. 31.

when that object passes the meridian. This time is the right ascension, and being multiplied by 15, gives the right ascension in degrees, &c.

The instrument by which the time of the transit over the meridian is accurately observed, is called the Transit Instrument (Art. 196), which is a telescope mounted upon an axis at right angles to the tube, which axis is horizontal and points due east and west, and in observatories is supported upon two stone piers, so that the telescope as it moves is always in the meridian (Fig. 31).

182. The declination and right ascension of celestial object, S, may also be obtained by observations made out of the meridian; thus if we observe the zenith distance ZS, and the azimuth, SZN, knowing the colatitude ZN we can calculate the side NS of the spherical triangle ZNS, that is the polar distance, and the angle ZNS, that is the hour angle of the star, and if we know then at the same time, by some means, the hour angle of the first point of Aries, or the sidereal time, we also get the difference of these, the right ascension of the object. The instrument used for observing the altitude and azimuth of an object, is called an Altitude and Azimuth Instrument. Its essential parts are a horizontal divided circle, which by the aid of a spirit level can be placed exactly horizontal, so as to represent the plane of the horizon ; in its centre it bears a vertical column, pointing towards the zenith, and bearing another circle parallel to it, and therefore representing a vertical circle. Attached to the axis of this circle, and revolving round it, is a telescope connected with an index, by which the direction of the telescope can be ascertained from the reading of the circle. The vertical column, which, together with the vertical circle and the telescope, revolves likewise on its axis, carries another index, which indicates the position of the column, and thus the direction of the vertical circle. If then the readings are known which correspond to the meridian and the zenith on the two circles, the two readings of the circles, when the telescope is directed to an object, give its zenith dis-

tance and azimuth. Usually, however, the observations of the azimuth and zenith distance are not used to determine the declination and right ascension of objects, as there are better and more direct methods for finding these, but rather to determine from the observations of stars, whose declination and right ascension are known, the zenith distance of the pole, or the colatitude, and the hour angle of the object, and from it the sidereal time. But at the Greenwich Observatory there is actually such an instrument in use for observing the moon, and which was specially built for this purpose, in order to multiply in a large ratio the observations which can be made from month to month on the moon. Although meridian observations are the best for ascertaining the position of heavenly bodies, yet the moon cannot be observed there some days before and after the new moon, in consequence of the proximity of that body to the sun. Besides this, at the moment of transit at other parts of the month, the observation is often rendered impossible by a clouded sky. An altitude and azimuth instrument of a special construction was therefore conceived by Mr. Airy, which allows the observations of the moon to be made whenever the sky is clear, and which by the advantages of its construction renders the observations as reliable as meridian observations.

183. There is also a method of obtaining both the declination and right ascension at the same time, by an instrument called an Equatorial Instrument. Although this, when well executed, is a very valuable instrument, yet it is too complicated to admit of the precision required in determining the places of heavenly objects, although very perfect instruments of this kind may be used to determine the differences of the places of objects which are not too far apart. In order to conceive an idea of such an instrument, we may suppose an altitude and azimuth instrument, mounted so as to make the formerly vertical axis become parallel to the axis of the earth, so that one of the circles is parallel to the equator, and the other perpendicular to it, representing there-

fore a declination circle. It is thus that all large telescopes are now mounted. It affords this advantage, that when the telescope is once directed towards an object, it can be followed as long as we please by a single motion—namely, by merely turning the whole instrument round on its polar axis; for the telescope remaining always pointed to the same polar distance, describes then an arc of a small circle in the heavens coincident with the star's diurnal path. With all the large equatorials a clock-work movement is used for turning the instrument round on its polar axis; and if its velocity is so regulated that it is exactly equal to the velocity with which the earth revolves on its axis, it is evident that when the telescope is directed towards an object, and the clock-work is going, it will follow exactly the diurnal motion of this object, and therefore keep it in the field of the telescope for whole hours in succession. This kind of mounting affords also the readiest means of bringing any object into the field of the telescope, as it is only necessary for this purpose to set the declination circle at the declination or polar distance of the object, and the other circle, which is called the hour circle, at the hour angle of the object, which is always the difference of the sidereal time, and the right ascension of the object. The observations made with such an instrument of the places of ob-objects are only of a differential nature. They are provided with micrometers, by which the *differences* of right ascension or declination between the object and a neighbouring fixed star are determined: the position of the latter is previously known, or may be observed at leisure.

184. The intersection of the ecliptic and equator not being marked on the concave surface, we must, for regulating the clock, make use of some fixed star, the right ascension of which is known: the clock may be put nearly to sidereal time, and the exact time being noted when a star, the right aseension of which is known, passes the meridian, the error of the clock will be known. Thus if the clock show $1^h\ 15^m\ 14^s$, when a star, the right ascension of which is $1^h\ 15^m\ 10^s$, passes,

the error of the clock will be 4^s, and every right ascension observed must be corrected by this quantity.

185. It is evident that the right ascension of some one star being known, the right ascensions of the rest may be obtained with much facility. The method which follows has been used by Mr. Flamstead, and by astronomers in general, to obtain the right ascension of α Aquilæ.

When the sun between the vernal and autumnal equinoxes has equal declinations, its distances in each case, from the respective equinoxes, are equal. We can ascertain when the sun has equal declinations, by observing the zenith distances for two or three days, soon after the vernal equinox, and for two or three days about the same distance of time before the autumnal, and then, by proportion, ascertain the precise time when the declinations are equal, assuming that the changes in right ascension are proportional for very short periods to the changes in declination: at these times also we can ascertain, by proportion, the differences of the right ascension of the sun and some star, by observing the differences at noon for two or three days, assuming that the *rate* of the sun's change in right ascension is uniform during the interval.

Let E = the right ascension of the sun, soon after the vernal equinox, then $180° - E$ = its right ascension before the autumnal, when it has equal declination.

A = the right ascension of the star in the former instance.

$A + p$ = the right ascension in the latter.

We obtain by help of observations $A - E$ and $(180° - E) - (A + p)$. Let these differences of right ascension be D and D', that is,

$$A - E = D$$

and $(180° - E) - (A + p) = D'$. From which we can determine E and A. For, adding these equations $180° - 2E - p = D + D'$ or $E = \dfrac{180° - (D + D') - p}{2}$ and thence $A = D + E$ is known. The value of p arises from the change of right ascension of the star in the interval

between the times of equal declinations, and is therefore known from the tables of precession and aberration, &c.

This kind of observation may be repeated many times for the same star between two successive equinoxes, and likewise in different years; and, by taking a mean of many results, great precision will be obtained.

The advantage of this method is, that, the sun's zenith distance being the same at the two times of observation, any error in the instrument will probably affect equally each zenith distance; and therefore we can exactly find when the declinations are the same, although we are not able to observe the declination itself with the greatest accuracy. In fact it is only the *change* in declination which we have to measure, which is independent of a knowledge of the latitude of the place or the sun's declination.

186. The construction of clocks for astronomical purposes has arrived at such a degree of perfection, that, for many months together, their rate of going can be depended on, to less than a second in twenty-four hours. This accuracy has been obtained by the nice execution of the parts, in consequence of which the errors from friction are almost entirely avoided, and, by using rubies for the sockets, and pallets, where the action is most incessant, the effect of wear is almost entirely obviated. But the principal source of accuracy is the construction of the pendulums, which are so contrived, that even in the extremes of heat and cold they remain of the same length. This is generally effected by a combination of rods of two different metals, differing considerably in their expansive powers. They are so placed as to counteract each other's effects on the length of the pendulum. Formerly brass and steel were used, the former expanding much more by heat than the latter. In this construction nine rods or bars were placed by the side of each other, and the pendulum, from its appearance, was called a gridiron pendulum. A composition of zinc and silver is now frequently applied, instead of brass, on account of its greater expansion, by which five

bars are made to serve. Other constructions are also used, for preserving the same length in the pendulum, but not so commonly. Mercurial pendulums are also of great use, which consist of a steel rod having at the lower extremity a vessel with mercury, the mass of the mercury being so adjusted that its expansion upwards neutralizes the effect of the expansion of the bar downwards, so that the length of the equivalent simple pendulum remains unaltered.

187. A clock of this description is absolutely necessary for an observatory. It is regulated to sidereal time, and the hours are continued to twenty-four, beginning when the vernal intersection of the ecliptic and equator is on the meridian; and not, like common clocks, at noon. But, however well executed the clock may be, it is depended on only for short intervals; the time it shows being examined by the transit of fixed stars, the right ascensions of which have been accurately settled. For this purpose the right ascensions of a number of stars have been determined with great exactness, and the places of these stars, called *standard stars*, are given for each ten days in every year in all Astronomical Almanacs. Several of these may be observed every day, each observation pointing out the error of the clock; and the mean of the errors will give the error more exactly. Nothing more, then, is necessary for determining the right ascension of a celestial object than to observe the sidereal time of its transit by the clock: that time, being corrected, if necessary, by observations of the standard stars, is the right ascension.

CHAPTER XI.

METHODS OF ASCERTAINING MINUTE PORTIONS OF CIRCULAR ARCS—ASTRONOMICAL QUADRANT—MURAL CIRCLE—TRANSIT INSTRUMENT—MERIDIAN CIRCLE—METHODS OF FINDING THE MERIDIAN.

188. As the arcs or limbs, as they are called, of astronomical instruments, are never divided nearer than to every five or two minutes, it is necessary briefly to explain the methods by which smaller portions may be ascertained: there are two methods now principally used, 1. by a *vernier*; 2. by a *microscope*.

189. THE VERNIER.—The first method is of more general use than the other, and is applied to a great variety of philosophical instruments, such as the Barometer. It is named after its inventor. It will be easily understood by an instance. Let the arc *lt* (Fig. 32)

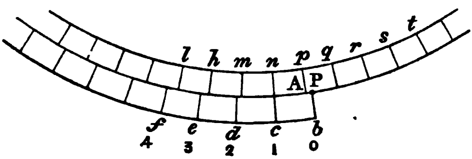

Fig. 32.

be divided into equal parts, *lh*, *hm*, *mn*, *np*, &c. each 20′, and let it be required to ascertain smaller portions, for instance, the distance of P from *p*A. Let another circular arc, called the vernier, 7° long, slide upon the arch *lt*, and let it be divided into twenty equal parts, that is, each part $= \frac{7 \times 60}{20} = 21'$. If these parts be *bc*, *cd*,

de, &c. then the division *d* coinciding with the division *m*, the division *c* will be $(21' - 20')$ or $1'$ beyond the division *n*; the division *b* $2'$ beyond the division *p*, &c. So that in this way we can ascertain portions of $1'$, $2'$, &c., although the arcs themselves are divided only into portions of $20'$. To apply this, suppose it were required to ascertain the distance of P from *p*A: let the vernier be slided till its commencement *b* coincides with P, then it will be seen what division of the vernier coincides with a division of the limb: the divisions of the vernier are numbered from its beginning 0, 1, 2, 3, &c. The number of the coinciding division of the vernier will, it is manifest, show the distance of the commencement of the vernier from the division on the limb or PA. In the application of this instrument to astronomical purposes, the vernier is so attached that its commencement, or point of Zero, as it is called, is always brought by the process of making the observation to the point from which the reading is to be made. In other applications, in the barometer, for instance, the commencement of the vernier is to be moved to that point.

This method of ascertaining the extent of small arcs is more frequently used where the measurement is only to be made to the nearest minute, but it may be readily applied to ascertain much smaller portions. Thus, if the limb be divided into portions of $20'$, and a vernier $= 19° 40'$ be divided into sixty parts, each of these parts will be $19' 40''$; and therefore an interval on the limb exceeds an interval on the vernier by $20''$, and so a space of $20''$ is ascertained.

Again, if the limb be divided into parts of five minutes each, and a vernier $= 4° 55'$ be divided into sixty parts, each of these parts will be $4' 55''$: and therefore an interval on the limb exceeds an interval on the vernier by $5''$.

190. If great accuracy is required in reading off a circle, a reading microscope is used for this purpose instead of a vernier. This consists of a compound microscope firmly placed over the limb and attached to some portion of the instrument which does not partici-

pate in the motion of the circle. The axis of the microscope is placed perpendicular to the limb, and a wire is placed in its focus so that it and the image of the division lines on the limb are distinctly visible together. The wire can be moved by a micrometer screw whose head is divided into 60 equal parts, and is read off by means of a little index. Supposing, then, the division lines on the limb to be two minutes apart, the distance of the microscope from the limb is so adjusted that exactly two revolutions of the screw carry the wire from one division to the next, so that, therefore, every part of the division on the screw head represents one second, and we may read it off by estimation to $\frac{1}{10}$ of a second. If we imagine, then, the screw to be turned so that its index reads zero, the position of the wire in this case corresponds exactly to the index P in the case of the vernier. A little round mark inside the microscope is placed so as to be bisected by the wire in this position, while other smaller ones are placed at the distance of 1 and 2 minutes. If, then, the circle is to be read off in a certain position, we look into the microscope and place the wire by turning the screw on the nearest division line on the limb which precedes the zero mark in the microscope. The reading of the head gives then the number of seconds, which must be added to the minutes read on the circle—only, if the zero mark should be over one minute distant from the nearest line on the limb, which is easily ascertained by the other marks in the microscope, one entire minute must be added to the reading of the circle.

191. The Astronomical Quadrant and Circle.—The quadrant for measuring zenith distances is moveable on a vertical axis, or fixed to a solid wall in the plane of the meridian. In the latter case it is called a mural quadrant. The telescope, which is moveable about the centre of the quadrant, has an index, usually a vernier, fixed to it, and moving on the divided arch of the quadrant. The plane of the quadrant being perpendicular to the horizon, and in the same vertical circle as the object, the telescope is moved till the object appears

near the centre of the field of view, touching or bisected by a wire, placed in the principal focus of the telescope, parallel to the horizon, or at right angles to the plane of the quadrant. The arc then between (o) on the vernier, and the lowest point of the quadrant from which the divisions commence ((o) of the arc), shows the zenith distance, provided the radius passing through (o) of the arc be vertical, and provided also that the line of collimation of the telescope be parallel to the radius passing through (o) of the vernier. The methods of ascertaining the exact place of the arc, pointed out by (o) on the vernier, have been shown in Art. 189, &c. The radius passing through (o) of the arc is generally made vertical, by help of a plumb line. The plumb line bisecting a point near the centre of the quadrant is made to bisect another point on the arc, by moving the quadrant in its own plane. These two points are placed by the maker parallel to the radius, passing through (o) of the arc.

192. *The line of collimation* of a telescope is the line joining the centre of the object glass, and the place of the image in the principal focus: this is the true direction of the object, in which it would be viewed by the naked eye. Hence it is evident that this line ought to be parallel to the radius passing through (o) on the vernier, that the angle measured by the distance of (o) on the vernier from (o) on the quadrant may show the angle contained by a vertical line, and the line of direction passing through the object, which angle is equal to the zenith distance of the object.

Thus OP (Fig. 33) represents the plumb line passing over two points. The line which joins these points is parallel to the radius CL, passing through (o) of the arch. The dotted line DI is the line of collimation parallel to the radius CV passing through (o) of the vernier;

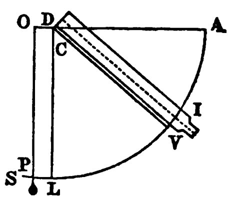

Fig. 33.

LV measures the zenith distance of the object, the

image of which is at I. The vernier being fixed to the telescope, the radius CV, while the telescope moves, always preserves the same relative position to the line of collimation. The position of the line of collimation must always be scrupulously attended to, and, if erroneous, must either be adjusted by moving the wire in the focus of the telescope, or the error allowed for; the latter is generally better, when the error amounts only to a small quantity.

193. To enable the observer to ascertain the error of the line of collimation in those quadrants that move on a vertical axis, the arc is continued several degrees beyond (o) (Fig. 33) as PS, and the zenith distance of the same object is to be observed with the arc of the instrument facing different ways. Thus, when a star near the zenith is observed, let CT (Fig. 34, *a.*) be the radius, parallel to the line of collimation of the telescope, CV the radius passing through (o) on the vernier. Then LV is the arc read off or observed; which is too little by TV. Let the quadrant be moved on its vertical axis half round: the position of the above lines will be as in Fig. 34, *b.* Then, that the telescope may be di-

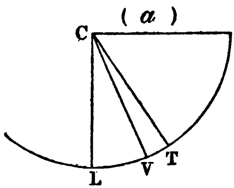

Fig. 34, *a.*

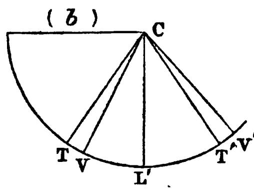

Fig. 34, *b.*

rected to the same star, it must be moved over the arc TT′, till it is parallel to its former position CT (Fig. 34, *a*), so that L′T′ = LT. The point V is transferred by the motion of the telescope to V′, &c. The arc now measured is V′L′ too great by V′T′ = VT. Hence 2VT (double the error of the line of collimation) = difference of the zenith distances of the same star observed in the two positions of the quadrant.

194. In modern times quadrants have been entirely superseded by circles, by which greater accuracy can be obtained than by the former, even when the diameter of the circle is far less than the radius of the quadrant. This is especially due to the fact that the error arising from the centre of the division not coinciding with the centre of revolution of the telescope to which the vernier is firmly attached, which is called the error of eccentricity, is entirely eliminated by the reading of two opposite verniers. We will suppose that C is the centre of the division, and K the axis of revolution of the telescope, and *a* and *b* the points of the circle read on the vernier when the telescope is directed towards the objects A and B. Then the angle which we wish to measure is BKA, while the angle read on the circle is *b*C*a*. But if there should be another vernier opposite the first, the points of the circle read on this vernier would be *a′* and *b′*—that is, we should read the angle *a′*C*b′* instead of *a′*K*b′*. But the one angle is as much too large as the other is too little, and, therefore, the mean of the readings of two opposite verniers is free from the error of eccentricity. It is for this reason that all circles are supplied with opposite verniers. Usually even there are four at equal distances, the mean of which is used so as to render the reading more free from any error of division. An entire circle has also the advantage that the line of collimation may be found by observing a star at any distance from the zenith, with the face of the instrument turned different ways, the change of the zenith distance of the star during the interval between the two observations being of course taken into account.

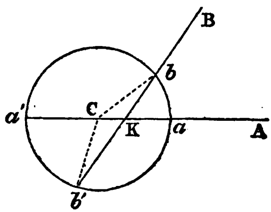

Fig. 35.

195. Mural Circle.—If such a circle is fixed to a solid wall in the plane of the meridian, it is called a mural circle. It is usually from four to six feet in

diameter, and revolves on a horizontal axis inserted in a stone pier. The reading microscopes, four, or sometimes six in number, are placed at equal distances, and firmly attached to the pier (Fig. 30). In order to make an observation with the mural circle, the telescope is directed towards a star at the time it passes the meridian, and the telescope is then moved in altitude until the star is exactly bisected by a horizontal wire inserted in the telescope. The circle and microscopes are then read off, but in order to obtain from this reading the zenith distance, it is necessary to know the zenith point on the limb of the circle. This is found either by observing a star direct as well as reflected from the surface of an artificial horizon or by determining the nadir point. In the first method a vessel with mercury is used, so placed that the image of the pole star[a] reflected from the mercury can be seen in the telescope. The telescope is then moved until the horizontal wire bisects the image and in this position the circle is read off. Afterwards the star is observed direct, and the circle read also in this position. As the surface of the mercury is horizontal, the depression of the reflected image of the star is as much below the horizon as the star itself is above it, and therefore the arc of the limb intercepted by the two readings, is equal to the double altitude of the star, while the mean of the two readings gives the horizontal point of the circle, and thus, by adding 90°, the zenith point. By subtracting this from the reading of the circle when a star is bisected by the horizontal wire of the telescope, we obtain the zenith distance, and, if the latitude of the place is known, also its polar distance.

In order to find the nadir point, the telescope is directed vertically downward, and a basin of mercury placed directly under it. An observer looking into the telescope sees then, if light is thrown upon the wire by a lamp, besides the horizontal wire, its reflected image, and when the telescope is moved so as to bring these

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[a] Because only stars near the pole remain sufficiently long in the field of the telescope.

two in coincidence, the telescope is directed exactly towards the nadir, and the reading of the circle gives therefore the nadir point, from which we obtain the zenith point by adding 180°.

In modern times mural circles have been superseded in many observatories by meridian circles, instruments first introduced in Germany, which are a combination of the mural circle and the transit instrument, and allow the observations formerly made with these two instruments, to be done by the same observer (Art. 199).

196. Transit Instrument.—The transit instrument, (Fig. 32), as before stated, is a telescope fixed at right angles to a cross axis. This axis is placed upon horizontal supports, upon which it turns. The instrument is to be so adjusted that the line of collimation, when the telescope is turned with its axis, may move in the plane of the meridian. In the principal focus of the object glass are placed five or seven wires, parallel to each other, and perpendicular to the horizontal axis. One or two wires bisecting the field of view are also usually placed at right angles to these.

In Fig. 36 the wires of a transit instrument are represented.

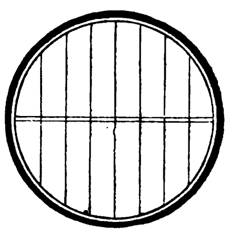

Fig. 36.

To make the centre wire *Cd* move in the plane of the meridian, three adjustments are necessary.

1st. To make the axis level. This is done by a spirit level placed upon the axis, the inclination of which can be corrected by adjusting screws attached to one of the supports of the axis. 2ndly. To make the line of collimation—that is, a line joining C and the centre of the object glass, perpendicular to the axis. This is done as follows :—Let the image of a distant object be bisected by the middle wire, and then take the axis off its supports, and reverse it; if the image is then bisected, when the telescope is again turned to the

object, the line of collimation is exact; if not, half the error must be corrected by moving the system of wires, and half by moving one of the supports of the axis. There is a provision for both these motions. The axis being again reversed, will verify the adjustment. 3rdly. The line of collimation is to be placed in the meridian. This is done by observing the transit of the pole star, above and below the pole. When the corrections for level and collimation have been accurately made, the middle wire of the telescope represents a vertical circle, and in order to represent also the meridian (that is, the vertical circle passing through the pole), it should exactly bisect the small circle which the pole star describes in 24 hours. The azimuth of the instrument must, therefore, be changed until the interval between the observed times of upper and lower culmination of the pole star is exactly equal to the interval between the lower and upper culmination. The adjustment is done by screws, which move one of the supports of the axis in the plane of the meridian. The adjustment may afterwards at times, when the pole star cannot be observed, be controlled by the help of a mark placed at a considerable distance, for instance at the distance of one or two miles, in the direction of the meridian of the centre of the instrument. The same mark will also serve for adjusting the line of collimation by the reversing of the instrument, as explained before.

197. The use of the transit instrument is to determine the right ascensions of the celestial bodies, and also the mean and apparent time. In observing the right ascension, the telescope is usually directed to the object by help of a divided semicircle, placed at one end of the axis, on which an index attached to, and perpendicular to the axis, consequently parallel to the line of collimation, moves; this index is to be set to the polar or zenith distance of the object, according as the semicircle shows polar or zenith distances.

This being done, the time of passage of the object over each wire is noted by the clock beating seconds, and

showing sidereal time, placed near the transit instrument. The mean of the observed times of passing each wire is to be taken to show more accurately the time at the middle wire. The time of passing each wire may be observed with great accuracy, because the telescope magnifies the diurnal motion, so that at one beat of the clock a star may be observed on one side of the wire as at *a*, and at the next beat at *b*. The eye is capable of pretty accurately proportioning the intervals *ac* and *bc*, so that the time may be noted to tenths of a second, and the mean from the five wires rarely deviates $\frac{2}{10}$ of a second of time from the truth, or 3″ of a degree. Thus right ascensions may be determined with nearly the same accuracy as zenith distances. For, as has been already shown, the time of the passage by the clock is the right ascension, provided the clock shows accurate sidereal time. This is seldom the case, and ought always to be examined by observing some of the standard stars before mentioned (Art. 187), the right ascensions of which have been determined with great accuracy.

198. The transit instrument serves also for finding the mean, and thence the apparent time.

If the sun, instead of appearing to move in the ecliptic with an unequable motion, appeared to move in the equator with an equable motion, in the period of its motion in the ecliptic, its return to the meridian would each day be later than the return of a fixed star by 3′ 56″ nearly; and a clock put to twelve o'clock, when the sun was in the meridian, would, if rightly adjusted, always continue to show twelve, when the sun, so moving, passed the meridian; and the time pointed out by the clock would be *mean time*.

The distance of an imaginary sun, so moving in the equator, from the vernal equinox, is equal to the mean longitude of the sun, or its mean distance from the vernal equinox; and this distance, reduced into time, is the right ascension of the imaginary sun. The mean longitude of the sun is given in the Solar Tables for the beginning of each year, and the mean motion in lon-

gitude, between the beginning of the year and each day, is also given. Whence the mean longitude is known, which, reduced into sidereal time, at the rate of 15° for 1 hour, gives the right ascension of the imaginary sun, after being corrected to reduce it to the true equinox. Hence, having the sidereal time, by a clock, or from the time shown by a clock corrected by observing the transit of a star, the mean time is readily found. For, the difference between the imaginary sun's right ascension at noon (the mean longitude of the sun converted into time), and the given sidereal time, is the sidereal time from noon: this is to be reduced into mean time, by diminishing it in the proportion of $24^h : 23^h\ 56^m\ 4^s.1$, or of 366 : 365 nearly. The mean time being found, the apparent time will be had by applying the equation of time, which will be explained hereafter.

199. A MERIDIAN CIRCLE is a transit instrument with a graduated circle of usually three feet diameter attached to its axis, or, still better, for the sake of symmetry, having two such circles, one on each side of the axis. These circles are divided from 0° and 360° into two minute spaces, and are read by micrometer reading microscopes, as described above, of which there are usually four on each side, which are firmly attached either to the two piers, on which the whole instrument rests, or to the supports of the axis, which are firmly let in, and cemented to the piers. An additional microscope at the level of the axis, which has a large field, taking in more than one degree of the limb, serves as a pointer for setting the instrument. In some instruments a small circle attached to the eye end of the telescope is used for this purpose. The arm bearing the vernier for reading this circle is furnished with a spirit level, and the zero point of the circle is placed so that when the telescope is directed towards the zenith, the bubble of the level is in the middle. If, then, the index is moved so as to read, for instance, 20°, and the telescope moved until the bubble is again in adjustment, it will be directed to a point 20° from the zenith, and thus this small circle serves

also as a pointer for setting the instrument at any zenith distance.

A meridian circle of this kind will shortly be mounted at the Dublin Observatory, in place of the transit instrument formerly in use. The focal length of the telescope is 8 feet, its aperture 6 inches, and the diameter of the circles 3 feet.

The woodcut (Fig. 36*a*) gives an idea of this instrument. The weight with which the axis of such an instrument (and also a transit) rests on its supports is almost entirely relieved by counterpoises on the top of the piers, which act on levers, whose hooks support the axis; the actual weight which presses upon the supports being thus reduced to about 15 or 20 pounds. The adjustments of the instrument are the same as those of the transit, and the zenith point of the circles is found by the same methods as those given before for the mural circle. When a transit of a star is being observed, the instrument is firmly clamped by a clamping apparatus attached to the axis, and when the star is near the middle wire the elevation of the telescope is slowly changed by the aid of a micrometer screw, until the star is bisected by the horizontal wire. The reading of the circle gives then, in connexion with the zenith point, the zenith distance, and thus also the polar distance of the star; while the mean of the observed times of transits over the wires leads to the knowledge of the right ascension of the star, as explained before.

200. Methods of Finding a Meridian Line.—The knowledge of the direction of the meridian is useful for several purposes, but absolutely necessary for adjusting a transit instrument. The first step, and that the most difficult, is to find it nearly: when this is done, it may easily be corrected by help of the transit instrument itself. Either of the two following methods, especially the second, will serve at once for finding it sufficiently near for most purposes, except for the transit instrument.

201. On a horizontal plane describe several concentric circles of a few inches in diameter. In the centre place

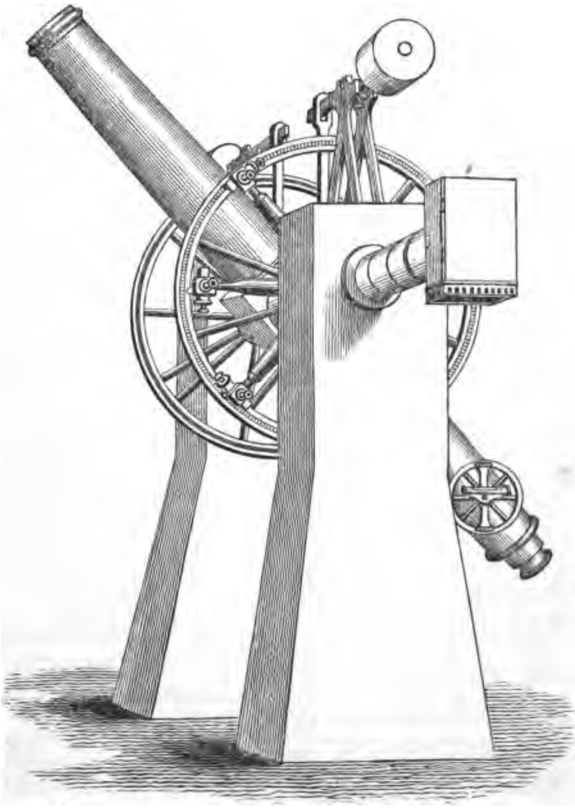

Fig. 36a, Meridian Circle.

a wire, a few inches long, at right angles to the horizontal plane. Note in the forenoon the point where the shadow of the top of the wire just reaches any of the circles, and watch in the afternoon the point where the extremity of the shadow again reaches the same circle. The arc intercepted between these two points being bisected by a radius, the radius will be in the direction of the meridian ; because the direction of the shadow is in the plane of the vertical circle passing through the sun, and the sun has equal azimuths at equal distances from noon, unless as far as the change of declination interferes.

This meridian may be transferred to any near place, by suspending a plumb line directly over the southern extremity of the line drawn as above, and noting when the shadow falls on that line: at this time another plumb line, suspended at the place where the meridian line is required, will, by its shadow, show the meridian.

The imperfections of this method of finding a meridian line arise from the inexact termination of the shadow, and from the change of the sun's declination in the interval of the two observations. The latter inconvenience is least in June and December, near the solstices.

202. In order to make this observation with more accuracy a quadrant, or circle, having an azimuth circle, is used, by observing equal altitudes on each side of the meridian, and thus bisecting the arc of the azimuth. If the sun be used, allowance must be made for the change of declination, and it is of course required that the vertical axis on which the instrument turns is made exactly vertical. It is, however, by no means necessary that the instrument have an accurate division, or in fact any division at all, as it is only necessary to see that the elevation of the telescope is the same for the two observations.

A good clock will serve instead of an azimuth circle, by observing equal altitudes of the sun or a star, half the interval of time corrected (if the sun is observed) will show when the object was on the meridian, and thence

the error of the clock will be ascertained, and so the time of the transit of any star may be computed, and the instrument adjusted at the time of that transit.

If the telescope is furnished with a vertical wire, the meridian can also be found by observing the pole star at the times of its greatest distance from the meridian. These occur at those points of its diurnal path in which the vertical circle is tangent to the diurnal circle, and therefore the pole star has at these times, for a short time, no motion in azimuth, but moves exactly upward or downward. If, then, at those times, the pole star is watched until it seems to move exactly along the wire, and the azimuth circle is read in the two positions of the telescope, the direction of the meridian is again found by bisecting the arc of the azimuth.

203. A portable transit telescope, turning on a horizontal axis, may also be used for determining the meridian. The deviation from the meridian of the telescope, approximately adjusted, may be found by observing the transits of a star near the zenith, and of the pole star. The transit of the former will give the sidereal time nearly; and comparing the time so found with the sidereal time given by the polar star, the difference, which may be considered as entirely the error from the pole star, will give the deviation from the meridian: for the deviation in seconds of a degree is to error in seconds of a degree of sidereal time of transit of pole star, as the sine of the polar distance of the pole star to the sine of the zenith distance. The reason of considering the whole difference, as the error of the pole star, is, that when the deviation from the meridian is small, the error of sidereal time from a star southward of the zenith is very small, compared to the error from the polar star. This is a very convenient method of approximating at pleasure to the meridian. Indeed, it may be used for the adjustment of a large transit instrument or meridian circle. The best method, however, for this purpose is that explained before, by comparing the times of continuance of a circumpolar star on the east and west sides of the meridian.

CHAPTER XII.

THE VELOCITY OF LIGHT, AND ABERRATION OF THE FIXED STARS AND PLANETS—THE EQUATION OF TIME—DIALS.

204. The velocity of light is the greatest velocity that has yet been ascertained. Astronomy furnishes two methods of measuring it. Without the discoveries in astronomy, the velocity of light would have remained unknown. The eclipses of Jupiter's satellites, and the aberration of the fixed stars, show us that the velocity of the reflected light of the sun, and the velocity of the direct light of the fixed stars, are equal.

205. The elder Cassini suspected, from observations of the eclipses of Jupiter's first satellite, that light was not instantaneous, but progressive. Roemer first fully established this fact, by a great variety of observations of the eclipses of the satellites of Jupiter.

Let the mean motion of a satellite be computed from two eclipses separated by a long interval, Jupiter being at each at its mean distance from the earth. Then an eclipse, when Jupiter is approaching conjunction, and therefore farther from the earth, happens later than is computed by the mean motion so determined. When Jupiter is in opposition, it happens sooner than according to the mean motion so determined.

From a great variety of observations, it appears that the velocity of light is such that, moving uniformly, it takes sixteen minutes to move over the diameter of the earth's orbit, or eight minutes in moving from the sun to us. This velocity is about 10,000 times greater than the velocity of the earth, which moves nineteen miles in a second. (Art. 112).

206. On the Aberration of the Fixed Stars and

Planets.—Another proof of the velocity of light is derived from the aberration of the fixed stars. The fixed stars appear, by observations made with accurate instruments, to have a small annual motion, returning at the end of a year precisely to the same place. A star near the pole of the ecliptic appears to describe about the pole a small circle parallel to the ecliptic; the diameter of this circle is 40″·9. Stars in the ecliptic appear to describe small arcs of the ecliptic 40″·9 in length. And all stars between the ecliptic and its poles appear annually to describe ellipses, the greater axes of which are parallel to the ecliptic, and equal to 40″·9. The axis minor is found by diminishing 40″·9 in the proportion of the sine of the star's latitude to radius. These phænomena cannot take place from the parallax of the annual orbit, because by it the latitude of a star would be greatest when in opposition to the sun, whereas then there is no aberration in latitude.

207. Dr. Bradley, who first discovered this apparent annual motion, when endeavouring to discover the parallax of γ Draconis, also first explained the cause of it. It arises from the velocity of the earth in its orbit, combined with the velocity of light.[a]

208. The application of a few mathematical principles enables us to explain and compute, with the greatest exactness, the laws of this phænomenon, which, although not the most striking, is perhaps one of the most pleasing objects of astronomical contemplation. The apparent irregularities of the motions of the different stars might for a long time have baffled the exertions of astronomers, had not the happy thought of applying the motion of light occurred to Bradley himself.

[a] Dr. Bradley's own account of this phænomenon is very interesting, and is found in the Phil. Trans. vol. 35. His observations were made with a zenith sector. In the present state of astronomy, an instrument, whether a quadrant or transit, that will not readily show the changes of the quantity of aberration must be considered as a very inferior instrument.

Let SA (Fig. 37) be the direction of light coming from a fixed star, and entering the telescope AD, carried in the direction DEF, by the motion of the earth. If the direction of the telescope be the same as the direction of the rays of light, it is clear that no ray can come to an eye at D, as from the motion of the telescope with the spectator they will be all lost against the interior of the tube. But if the tube be inclined in the position DB, so that BE : DE : : vel. of light : vel. of the earth, then a ray SB parallel to SA entering the tube at B, will pass through the axis of the tube in motion, and be seen by the eye arrived with the telescope at E, while the light is passing from B to E. The ray of light will be always found in the axis of the telescope, carried by the motion of the earth, parallel to itself. The telescope being in the position EC, the star is judged to be in that direction, although it be actually in the direction EB. Hence BEC is the angle of aberration, and the aberration is always toward that part of the heavens to which the earth is moving. As BE is above 10,000 times greater than DE, it follows that the angle DBE must be very small, and therefore its equal BEC, the aberration must be very small. It is evident that DBE, and therefore BEC, is a maximum when BDE is a right angle, because sin. DBE : sin. BDE : : DE : BE : : vel. earth : vel. light, a given ratio. Therefore, when sin. DBE is greatest, the sin. BDE is greatest—that is, when BDE is

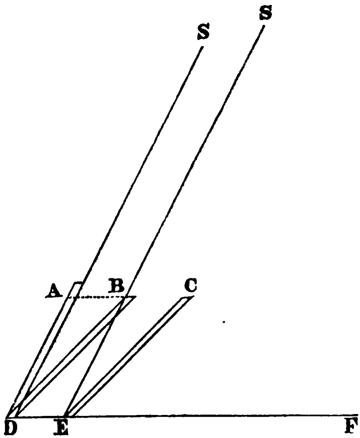

Fig. 37.

a right angle. Then vel. of light : vel. of earth : : sin. 90° : sin. of greatest aberration, and therefore sin. of greatest aberr. $= \frac{\text{vel. of earth}}{\text{vel. of light.}} =$ sine 20″·45 nearly. In general the sin. of BEC (or since BEC is very small) the aberration $= \frac{\text{vel. of earth}}{\text{vel. of light}}$. sin. BDE or 20″·45 × sin. of the earth's way (the angle between the directions of the earth's motion and of the star).

209. It may illustrate this matter to consider the earth at rest (which may be done by giving to the earth and to the rays of light motions equal and opposite to that of the earth), and the light from the star to have motions in two directions, viz. the actual velocity of light in the direction BE, and another in a direction parallel and opposite to the earth's motion DE; the motion of the light will be then in BD the diagonal of a parallelogram, of which the sides are BA equal and opposite to DE, and BE.

To the naked eye the sensation must be the same, whether the light strikes the eye with a motion in the direction ED, or the eye strikes the light in the opposite direction; and therefore we may consider the light meeting the eye as coming in a direction compounded of two motions, that of light, and that of the earth, and therefore the same aberration takes place as in a telescope.[a]

210. The direction of the earth's motion is always towards the point of the ecliptic 90° behind the sun, because the earth's motion is in a tangent to its orbit which makes a right angle with the direction in which the sun appears, the orbit being considered to be circular.

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[a] The effect of the joint motions of light and of the earth is commonly illustrated by the apparent motion of drops of rain falling vertically. If a man moves forward in such a shower they appear to be falling against his face in a slanting direction; or if a shot is fired from a battery against a ship moving at right angles to the direction of the shot, the line joining the shot holes will be inclined to the keel, and the shot will appear to pierce the ship as if it came from a battery slightly a-head.

Hence since the stars appear to be thrown forward in the direction of the earth's motion, they all aberrate toward this point of the ecliptic, from which consideration the general phænomena of the aberration may be easily understood.

The phænomena of the aberration may be also thus shown in a different way:

Conceive a plane passing through the star, parallel to the plane of the earth's orbit, and a line in this plane, parallel to the direction of the earth's motion, the length of which is to the star's distance as the velocity of the earth to the velocity of light, the extremity of this line will be the place in which the star appears. Now we may consider, without sensible error, the orbit of the earth as circular, and its velocity as uniform; therefore this imaginary line drawn from the star, parallel to the tangent to the earth's orbit, will be always of a constant length; and as the tangent in the course of a year completes a revolution, this imaginary line will also, in the course of a year, complete a revolution, and its extremity describe a circle about the star. To a spectator on the earth, the star, in the course of a year, will appear to describe the circumference of this imaginary circle, the plane of which is parallel to the plane of the earth's orbit: and he will orthographically project this circle on the concave surface, by which it will appear an ellipse.[a] To find the axis major of this ellipse, we are to consider that the diameter of the circle of aberration, perpendicular to a secondary to the ecliptic passing through the star, will be projected into the axis major of the ellipse. When the earth, seen from the sun, is in this secondary to the ecliptic, the line joining the star and earth,

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[a] Or the lines drawn from the earth to the star in the course of a year, being on account of the star's enormous distance taken to be the same as those drawn from the sun to its several positions, will describe an oblique cone on a circular base, which will be cut by the celestial sphere in an ellipse, the semiaxis major of which is $20''.45$, and the semiaxis minor $20''.45$ sin. star's lat.

being perpendicular to the tangent to the earth's orbit, will be at right angles to the direction of the earth's motion, and therefore the aberration will be then greatest, and equal to $20''\cdot45$ (Art. 209). Hence the semiaxis major of the ellipse is $20''\cdot45$. The star's longitude is most increased when the star's and sun's longitudes differ by 180°, and most diminished when the longitude of the sun is the same as that of the star, the aberration being in these cases a maximum, for the above reason, and takes place altogether in longitude. When the sun's longitude exceeds that of the star by 90°, the radius of the circle of aberration being perpendicular to the direction of the sun as seen from the earth, is in the plane of the star's circle of longitude, and being parallel to the ecliptic, is diminished by projection on the concave surface, in the proportion of the cosine of the angle between the plane of the ecliptic and the plane perpendicular to the line joining the earth and star, or, of the sine of the star's latitude, to unity. The radius of the imaginary circle, thus diminished, becomes the semiaxis minor of the ellipse. The star's latitude is most diminished when the sun's longitude exceeds that of the star by 90°, and most increased when the sun's place is 90° behind the star, the aberration in these cases being altogether in latitude.

When the star is in the ecliptic, it is evident that the imaginary circle of aberration must be projected into a right line, or rather an arc of $40''\cdot9$. A star in the pole of the ecliptic appears to describe a circle $40''\cdot9$ in diameter, because the imaginary circle is not changed by projection. In practice it is necessary to compute the effects of aberration in right ascension and declination.

211. The aberration of a planet is somewhat different from that of a star; for if the planet's motion were equal and parallel to that of the earth, no aberration would take place. From the small velocity of the moon about the earth, compared with the velocity of light, no sensible aberration takes place with regard to its velocity about the earth; and the moon and earth being carried

together round the sun with nearly the same velocities, no aberration from thence occurs in the place of the moon.

The best method of finding the aberration of a planet or comet, is by first considering the effect of the earth's motion on the apparent place: this is the same as for a fixed star; and then the aberration arising from its own motion; this is readily computed; for the planet, supposing the earth at rest, appears not in the place which it occupies when the light reaches the eye, but in the place it was in when the light left it, and therefore, by combining these, it is easy to see that it is only necessary to compute the place of the planet for a time, so much earlier by the time that the light is coming from the planet to the earth; this will be the planet's place at the instant of observation.

212. The velocity of light determined by the eclipses of Jupiter's satellites has been considered as exactly the same as that determined by the aberration of the fixed stars.[a]

As we are certain of the velocity of light by the eclipses of Jupiter's satellites, and also that the consequence of that velocity, and of the velocity of the earth, must be an aberration in the fixed stars, we have, from the observation of the aberration, an independent proof of the motion of the earth. In recent times other means of measuring the velocity of light have been discovered by M. Fizeau, and M. Fourcault has obtained results

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[a] The maximum of aberration deduced from the velocity of light, as determined by the eclipses of Jupiter's satellites, appears to be 20″ 25. Bradley's observations appear to give the same quantity; but Bradley himself, on a revision of his observations. fixed it at 20″. But Dr. Brinkley's observations, made at the Observatory of Trinity College, Dublin, with the 8-feet circle, give it so great as 20″·80. M. Bessel, from Dr. Bradley's Greenwich observations, makes it 20″·71. Lindenau, from observations of the pole star in R. Ascension, makes it 20″·45. M. Struve, from observations in Right Ascension, makes it 20″·60. It appears, therefore, highly probable that it equals 20″·45. By continuing the observations, it is hoped, greater certainty will be obtained in this important element.

agreeing with those derived from former methods as above given.

213. Equation of Time.—The rotation of the earth on its axis is among the few perfectly equable motions known; the period of which, or 24 hours of sidereal time, might serve as a measure of duration; but this is not convenient for the purposes of civil life. For these, the period of a solar day, or the interval elapsed between two successive passages of the sun over the meridian, is a much more convenient measure of time. But this interval is variable, for it is greater than the time of the earth's rotation by a variable quantity. This variable quantity is the time the hour circle passing through the sun takes to move over an arc equal to the increase of the sun's right ascension during a solar day. Now the daily increase of the sun's right ascension is variable from two causes, viz., 1st, the inclination of the ecliptic to the equator; and 2nd, the unequal apparent motion of the sun in the ecliptic arising from the unequal motion of the earth in its orbit. It is evident that the sun's increase of right ascension must be variable, on account of the obliquity of the ecliptic to the equator; because, when the sun is in Aries, its motion being oblique to the equator, the rate of increase of right ascension must then be less than the rate of increase of longitude; when at the tropics, its motion is parallel to the equator, and being nearer the pole of the equator than the pole of the ecliptic, the arc of the equator between two successive circles of declination will be greater than the arc of the ecliptic, and, consequently, its motion in right ascension must be then greater than its motion in longitude.[a] Hence it is evident that the length of a solar day must be variable, and consequently that the time, which is called

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[a] It is not difficult to prove that the rate of increase of the sun's right ascension is, to the rate of increase of its longitude, as unity multiplied by the cosine of the obliquity of the ecliptic is to the square of the cosine of the sun's declination. The last term decreases from the equinox to the solstice, and therefore the first must increase. If S and S′ be two consecutive positions of the sun in the ecliptic, and if great circles be drawn from the North Pole, P, through each, to cut

apparent solar time, or apparent time measured by a solar day and parts of a solar day, must require a correction, which is called the Equation of Time. The perfection of the mechanism of a clock depends on the uniformity of its motion; therefore a clock intended to shew solar time, must be regulated according to mean solar time, and the equation of time must be allowed in deducing apparent time from the time shewn by a clock, or in setting a clock from the position of the sun in the heavens, as shewn by a sun-dial, or by an astronomical instrument.

214. If the sun, instead of moving as it does unequally in the ecliptic, moved uniformly in the equator, the interval between two transits of the sun over the meridian would then be always the same, and would be an exact measure of time. Let us suppose, then, an imaginary sun moving uniformly in the equator in the same time in which the sun appears to move in the ecliptic, and having its right ascension, or distance from the beginning of Aries, equal to the mean longitude of the sun, (or the longitude of the sun supposed to move uniformly in the ecliptic). The time measured by this imaginary sun so moving, is called *mean solar time*, or mean time. The hour circle passing through the imaginary sun describes 360 degrees in 24 hours mean time, and that through the real sun the same in 24 hours apparent time, therefore each describes 15 degrees in an hour.[b]

215. The difference between mean and apparent time, i. e., the equation of time, is evidently equal to the differ-

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the equator in the points T, T', and a small circle be drawn through S, parallel to the equator, meeting S' T' in X, then the change of longitude is SS' and the change of R A is $SPS' = \frac{SX}{\text{Sin } PS} = \frac{SX}{\text{cos. dec.}}$ but $SX = SS' \sin SS'X$, that is (from Spherical Trigonometry) $= SS' \cdot \frac{\text{cos obliquity}}{\text{cos decl.}}$ $\therefore$ the change in RA, $= SS' \frac{\text{cos obliquity}}{\cos^2 \text{decl.}}$.

[a] Art. 198.

[b] The greatest difference between 24 hours mean time and 24 hours apparent time is 30^s.

ence between the right ascension of the sun and the mean longitude of the sun, converted into time at the rate of 360° for 24^h, or 15° for 1 hour.

The sun's *mean* longitude is given by the solar tables, and thence the *true* longitude is calculated: by the latter, and the obliquity of the ecliptic, the right ascension may be computed, from the formula of spherical trigonometry, cos. ω (obliquity) = tan RA·cot long., and then the difference[a] of mean longitude and right ascension, converted into time at the rate of 15° to an hour, is the equation of time. Hence the computations for finding the right ascension of the sun will also serve for finding the equation of time.

216. In order to consider the changes in the equation of time during the course of a year, we will consider separately the effects produced by the obliquity of the ecliptic and the unequal motion of the sun in longitude.

Equation of time caused by the obliquity of the ecliptic alone. In this case we suppose the sun to move equably in the ecliptic, and its longitude to be proportional to the time from the vernal equinox; it is evident that between the vernal equinox and the summer solstice the imaginary sun moving in the equator will be to the east of the foot of the secondary to the equator passing through the sun in the ecliptic, and, consequently, this secondary will come up to the meridian before the imaginary sun in the equator, or the apparent noon will be before the mean noon, and if m and a be the mean time and the apparent time, $m - a$, the equation of time, will be *negative;* at the solstice they will coincide and $m - a = o$: from the summer solstice to the autumnal equinox, for a similar reason

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[a] Accurately the equation of time is the difference between the sun's right ascension (a), and mean longitude (m) reckoned on the equator from the true equinox, because the right ascension is computed from the true equinox. By the sun's mean longitude, reckoned on the equator from the true equinox, is meant the sun's mean longitude always reckoned from the mean equinox) corrected for the equation of the equinoxes in right ascension.

the sun in the equator will be to the west of the above-named secondary, and therefore will come up to the meridian before it, and consequently $(m - a)$ will be *positive;* at the autumnal equinox they will coincide, and $m - a$ will vanish; by a similar mode of reasoning it will be seen that from the autumnal equinox to the winter solstice $(m - a)$ will be *negative*, and from the winter solstice to the vernal equinox *positive*.[a]

Equation of time caused by sun's unequal motion in longitude. It is found by observation (Art. 83), that the motion of the sun in the ecliptic during the year, from day to day, is unequal; its daily value will be found to vary from 2′ 33″ to 2′ 23″ in an hour, while its apparent diameter (which is inversely proportional to its distance from the earth) varies from 32′ 34″ to 31′ 29″. It is also observed that at *any time* its motion in an hour is proportional to the square of its apparent diameter at the time, and therefore inversely as the square of its distance from the earth; consequently, if lines be drawn from the sun to the earth on two consecutive days they will, with the space described by the earth, form a triangle (nearly isosceles with the vertical angle small), and the area of this triangle being ½ the square of the distance from the sun, multiplied by the angle at the sun between the two distances (or the change in sun's longitude during the day), will be constant. This is Kepler's *second law* of the planetary motions, and is true of all the planets as well as of the earth. Kepler also discovered that the earth and all the planets, instead of describing concentric circles round the sun, described ellipses more or less differing from a circle with the sun in

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[a] To find when $m - a$ is greatest.—By Art. 215, cos ω (obliquity) $= \dfrac{\tan a}{\tan m} = \dfrac{\sin a . \cos m}{\sin m . \cos a} \therefore \dfrac{1 - \cos\omega}{1 + \cos\omega} = \dfrac{\sin m \cos a - \sin a \cos m}{\sin m \cos a + \sin a \cos m} = \dfrac{\sin(m - a)}{\sin(m + a)}$; or $\sin(m - a) = \sin(m + a) \tan^2 \tfrac{1}{2}\omega$. Hence $(m - a)$ is greatest when $m + a = 90°$; and its greatest value is given by the equation $\sin(m - a) = \tan^2 \tfrac{1}{2}\omega$; this will give $m - a = 2° \, 28' \, 20''$, or 9 min. 53 sec. in time, for a max. value.

their common focus. It is observed that the earth is nearest to the sun, or in *perihelion*, on December 31st, and farthest from the sun, or in *aphelion*, on July 1st. These points are called the *apsides* of the earth's orbit, because the motion is then strictly perpendicular to the line joining the sun and earth, and the line joining these points is called the *line of apsides*, or the axis major of the orbit. Now, since the apparent motion of the sun in a day is inversely proportional to the square of its distance from the earth, it will be the greatest when the earth is in *perihelion*, or at the beginning of January, and least when in *aphelion*, or at the begining of July.

Let us suppose an imaginary sun moving *in the ecliptic* at the uniform rate of 2′ 28″ in an hour, to start together with the true sun from perihelion, the true sun will gain upon the mean in consequence of the distance being less than the average, and consequently the mean will be to the west of the true, and will come up to the meridian before the true; or $m' - a'$ (if m' be the mean time and a' the apparent time) will be positive; this will go on increasing until about the 1st of April, when it will be a maximum (7 minutes of time), and after that time it will decrease until the 1st of July, when the true sun and mean sun will meet again at aphelion, and $m' - a' = o$; after this, from the distance of the earth from the sun being greater than the average, the true motion in longitude will be less than the mean, and the true sun will be to the west of the mean, and will come up to the meridian before it, and $m' - a'$ will be negative until the next perihelion.

In order to bring the effects together which are produced by these two causes acting simultaneously, let us graphically represent the effects produced by each, and add them together. Let us draw a vertical line AB (Fig. 37), and divide it into twelve parts corresponding to the months, and at each point corresponding to a day of the year draw a perpendicular to it, representing in length the equation of time, from the first of

these causes on that day, measuring it to the right of the line when it is positive, and to the left of the line when negative, and connect the extremities of these perpen-

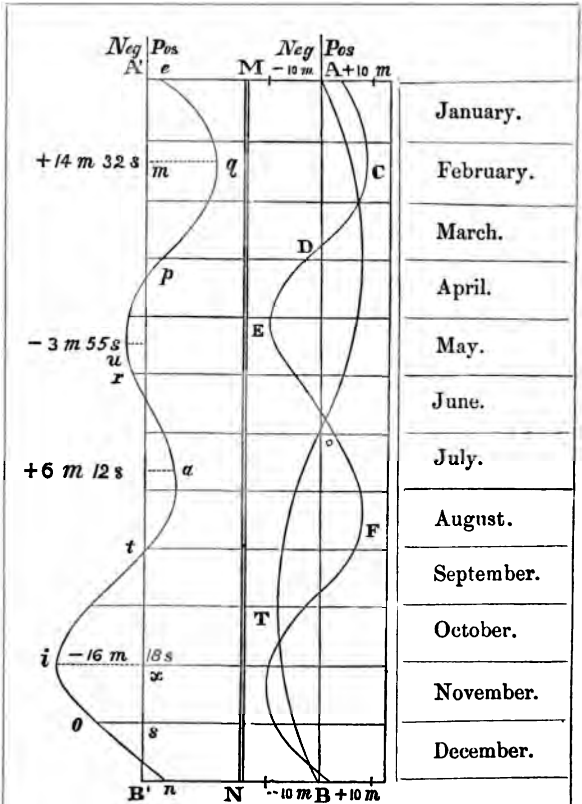

Fig. 38.

diculars by a curved line ACDEF. Let us also erect at each point a perpendicular representing the equa-

tion of time from the second cause, and in like manner form the second curve AOTB, then the joint effect will be found for any day by taking the algebraic sum of these perpendiculars with their proper signs. And the whole equation of time will be represented graphically by the curve *eqpuation* to the left of MN, constructed by erecting at each corresponding point of A′B′ a perpendicular equal to this algebraic sum, the maximum positive value *mq* being + $14^m\ 32^s$ on the 11th February, and the maximum negative *xi* – $16^m\,18^s$ on the 2nd November. It will vanish four times in the year, when the curve crosses A′B′, or on April 15, June 14, September 1, and December 25; there will be also a second but smaller positive maximum of + $6^m\ 12^s$ on July 26, and a second smaller maximum of – $3^m 55^s$ on May 14.

A more particular consideration of the equation of time would be useless here. Indeed everything of consequence may be considered as explained, when it is said to be equal to the difference, converted into time, between the sun's true right ascension and mean longitude, corrected for the equation of equinoxes in right ascension.

It is to be observed that the circumstances of the equation of time will change, with a change in the longitude of the earth's aphelion, which moves forward from the equinox at the rate of 1′ 2″ in a year. The longitude at present, as seen from the sun, is $9^s\ 9°\frac{1}{2}$. About 4000 B. C. (the supposed time of the creation) it coincided with the place of the earth at the vernal equinox.

The time shown by a dial is apparent time, for it is the angle between the hour circle passing through the sun and the meridian, converted into time.

217. On Dialling.—In a dial, the shadow of a straight line, by its intersection with a given plane, points out the apparent hour. The line by which the shadow is made is called the stile or gnomon. Let a meridian line be drawn on a horizontal plane (Art. 201, &c.), and on this plane a gnomon or stile fixed, making an angle with

the meridian line equal to the latitude of the place, and being also in the plane of the meridian. This gnomon then will be in the direction of the celestial axis (Art. 39), the shadow therefore will always be in the plane of the hour circle in which the sun is, and because the sun is always in the same hour circle at the same distance from noon, whatever be its declination, it follows that the intersection of the shadow and horizontal plane is always the same at a given hour. Therefore these intersections of the shadow being marked, will always serve for pointing out the hour from noon. These intersections are called hour lines of the dial, and a dial thus constructed is called a horizontal dial. The angles which these hour lines make with the meridian may be determined as follows:—let S be the position of the sun at 1 o'clock, P.M., then drawing through S and the North Pole P a great circle SPH to meet the horizon at H, the shadow of the gnomon corresponding to 1 o'clock will point to H; now, if N be the north point of the horizon, in the right-angled spherical triangle HNP, tan. HN, = sin. PN (lat.) × tan. HPN (= hour angle from noon or 15°); for 2 P.M., H′PN would be 30°, and so on.

Thus the angle which any hour line makes with the meridian may be found, and a horizontal dial constructed.

If a vertical plane, facing the south, at right angles to the meridian, be used, the intersections of the shadow and this plane, or the hour lines of the dial, will be found, by computing the distances of the hour circles from the meridian on the prime vertical. A dial so constructed is called a vertical dial.

It is evident that the plane of the dial may make any given angle with the prime vertical, and the hour lines be readily computed by a spherical triangle. When the plane of the dial faces the east or west, the stile is placed at a distance from, and parallel to its plane, because the plane of the dial is itself in the plane of the meridian.

CHAPTER XIII.

APPLICATION OF ASTRONOMY TO NAVIGATION—HADLEY'S SEXTANT—LATITUDE AT SEA—APPARENT TIME—VARIATION OF THE COMPASS—LONGITUDE AT SEA.

218. THE uses of astronomy in navigation are very great. It enables the seaman to determine by celestial observations his latitude and longitude, and thence discover his situation with an accuracy sufficient to direct him the course he ought to steer for his intended port, and to guard him against dangers from shoals and rocks. It also enables him to find the variation of his compass, and so affords him the means of sailing his proper course.

Almost all the astronomical observations made at sea consist in measuring angles, and the difficulty of taking an angle at sea, on account of the unsteady motion of the ship, is sufficiently obvious. In taking an altitude, the plumb-line and spirit-level are entirely useless. In observing the angular distance of two objects, the unsteadiness of the ship makes it impossible to measure it by two telescopes, or by one telescope successively adjusted to each object.

219. These difficulties were soon seen when nautical astronomy began to be improved. Many attempts were made to invent a proper instrument. The ingenious Dr. Hooke proposed several methods. Many years afterwards Mr. Hadley proposed the instrument called Hadley's quadrant, now, however, usually called Hadley's sextant, for a reason that will be mentioned. A few years after Mr. Hadley's invention was communicated to the world, a paper of Sir Isaac Newton's was found, describing an instrument nearly of the same construction. The principle of this invaluable instrument

is, that in taking the angular distance of two objects, the image of one of them, seen after two reflections, coincides with the other object seen directly; and this coincidence is in nowise affected by the unsteadiness of the ship. The operation by which the coincidence is made, measures the angular distance of the objects.

220. HADLEY'S SEXTANT.—Let A and B (Fig. 39) be two celestial or very distant objects; HO, IN the sec-

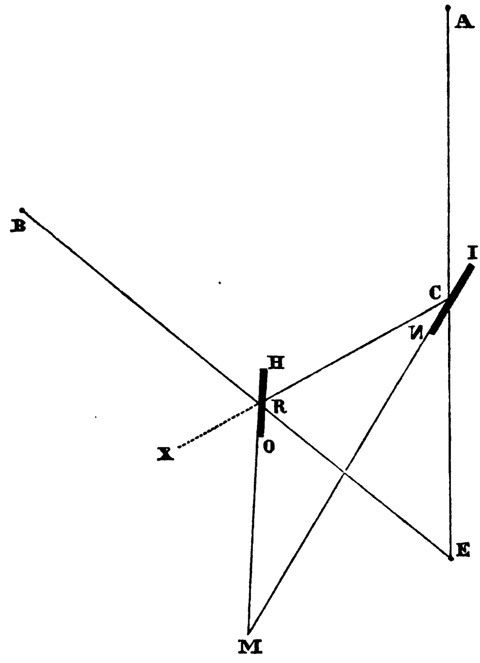

Fig. 39.

tions of two plane mirrors, in the plane passing through the objects and eye. The mirrors are supposed to be perpendicular to this plane. Let a ray of light, AC, from the object A, incident on the mirror IN, be re-

flected in the direction CRX, and so be incident on the mirror HO, from whence it is again reflected in the direction RE, coinciding with the direction of a ray, BR, from the other object, B. Then an eye anywhere in the direction of the line RE, will see the object A, coincident with the object B, if a portion of the mirror immediately above the section HO be transparent. Thus we may make two distant objects appear to coincide by a proper position of the mirrors, viz., by inclining the mirrors at an angle equal to half the angular distance of the objects. For produce the sections of the mirrors to meet in M, and produce AC to meet BRE in E. Then by the principle of the equality of the angles of incidence and reflection, the angle ERX is bisected by the mirror HO, and the angle RCE by the mirror NI, consequently the angle $M = XRM - RCM = \frac{1}{2} XRE - \frac{1}{2} RCE = \frac{1}{2}(XRE - RCE) = \frac{1}{2} REC$, or the angular distance of the objects equals twice the inclination of the reflectors. Hence if we move the reflector IN, so that both objects may appear to coincide, and can then measure the inclination of the reflectors, we shall obtain the angular distance of the objects. This principle is used in Hadley's sextant as follows.

221. ACB (Fig. 46) may represent the sextant. The angle ACB is 60°, but the arc AB extends a few degrees beyond each radius. A moveable radius, CV, called the index, revolves

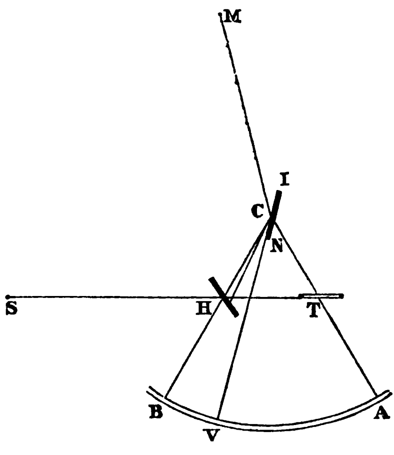

Fig. 40.

about the centre C, carrying a plane mirror, IN, perpendicular to the plane of the sextant, which mirror faces another mirror, H, also perpendicular to the plane of the sextant. This latter mirror is fixed with its plane parallel to CA, the position of the mirror IN, when the radius CV passes through zero or (o) of the arc. The upper part of the mirror H is transparent, through which, by help of a telescope fixed at T, parallel to the plane of the sextant, the object S may be seen directly, while the image of M, seen by reflection, appears to touch it. The angular distance of the objects M and S is then, as has been shown = twice the inclination of the mirrors H and IN = (because H is parallel to CA) 2 VCA. Hence the degrees, minutes, and seconds in VA, shown by a vernier, attached to the extremity of the index, would give half the angular distance of the objects; but as the arc VA is only half the angular distance of the objects, for convenience each degree, &c., is reckoned double; thus if VA be actually 42°, it is marked 84°, &c.

The mirror IC is called the *index glass*, and H the *horizon glass*, because in taking the altitude of the sun at sea, the horizon is seen directly through this glass.

In most sextants there is a provision for adjusting the plane of the horizon glass, parallel to the radius passing through zero of the arc, or rather parallel to the plane of the index glass, when the index is at zero of the arc. This is done by making an image coincide with its object seen directly, when the index passes through zero. Or the quantity of the error may be determined by measuring a small angle, for instance the sun's diameter, on each side of zero of the arc. Half the difference is the error of the index, and it is most convenient to allow for this, as it cannot be corrected so exactly as its quantity can be ascertained.

222. The best instruments, intended for taking the angular distance of the moon from the sun and stars, are made with great exactness. The radius of a sextant varies in length from five to fourteen inches. The usual

length is about ten or twelve inches, and these admit of measuring an angle to 10″ or less, by help of the vernier. Ordinary instruments are also made, merely for taking altitudes. Plain sights are only used with these, and they are seldom adapted to take altitudes nearer than two or three minutes.

As an altitude is never greater than 90°, it is evident, for an altitude, a greater arc than 45° is not required. The instruments, therefore, made only for taking altitudes should properly be called *octants*, instead of quadrants, as they are sometimes named. The angular distance of the moon from a star is sometimes measured when 120°, for such distances an arc of 60° is necessary, and therefore the instruments intended for the longitude at sea are called *sextants*.

223. The celebrated Mayer, whose lunar tables have been mentioned, recommended a complete circle for measuring the angular distance of the moon from the sun or stars by reflection, as in Hadley's instrument. Some of the advantages proposed were similar to those of the astronomical circle over the astronomical quadrant; also by making the horizon glass moveable, the same angle could be repeated on different parts of the limb, and by repeating the angle many times, and taking a mean, the errors of division were almost entirely done away.

224. Let us proceed to the application of the sextant for finding the latitude, apparent time, variation of the compass, and longitude at sea.

The latitude at sea is most readily and usually found by observing the meridian altitude of the sun, and correcting it for refraction as in table page 43. At sea the horizon is generally well defined. The sextant being placed in a vertical position, the upper or lower limb of the sun, by moving the index, is brought down to the horizon seen directly. The index shows the altitude; but it must be noted, that as the eye of the spectator is elevated above the level of the sea, the apparent altitude is to be diminished by the depression of the horizon,

called the dip[a]. The sun is known to be on the meridian when it ceases to rise higher, or when the index angle ceases to increase, and for this purpose the observations commence half an hour before noon, and are frequently repeated. An error of one or two minutes is of little consequence in finding the latitude at sea, as it makes only an error of one or two miles in the place of the ship. Oftentimes the horizon is not sufficiently defined to attain to great accuracy. A star can seldom be used, on account of the horizon not being sufficiently visible, but the moon oftentimes may. The correct meridian altitude of the sun's centre and the declination being known, the latitude is easily found, being always equal to the sum (or difference) of the zenith distance and declination, according as the object is north or south of the celestial equator.

225. It often happens that it is cloudy at noon, and therefore an observation cannot be made: this sometimes is the case for several days together, when perhaps the sun is occasionally seen during that time. The latitude in such circumstances may be obtained by observing two altitudes of the sun, and noting the interval of time between, by a good watch: from this data and the sun's polar distance the latitude may be found. Let P be the pole, Z the zenith, S and S′ the two positions of the sun, we know PS, PS′ and SPS′, consequently the base SS′ and the base angle PSS′ (by spherical trigonometry),

[a] The following table showing the dip for different elevations of the eye, allowing $\frac{1}{10}$th for terrestrial refraction, is given by Professor Loomis.

Height.	Depression.	Height.	Depression.
5 feet.	2′ 9″	30 feet.	5′ 15″
10 ,,	3 2	50 ,,	6 46
15 ,,	3 42	70 ,,	8 1
20 ,,	4 17	100 ,,	9 35

The square of the tangent of the dip is evidently twice the height, divided by earth's radius.

we know then of the spherical triangle ZSS′, the three sides, and we can calculate the angle ZSS′; having as before PSS′ we have their difference PSZ; and of the triangle PSZ, we have PS, SZ, and the angle PSZ to find the co-latitude ZP. This requires the solution of three spherical triangles.

It may be mentioned, once for all, that it is here only intended to give a general account of the observations necessary for nautical purposes. The particulars of the methods of computation are to be found in the different works on practical astronomy or navigation.

226. *The apparent time may be found at sea*, by observing the altitude of the sun. If we note the time when it is on the meridian, as in Art. 224, we have the apparent noon, and by adding or subtracting the equation of time we have the mean time. But the time may be calculated by an observation of the sun's altitude at any time. For knowing the latitude of the place and the sun's declination, we have the three sides of a spherical triangle, viz., the sun's zenith distance, the polar distance, and the co-latitude of the place, to find the hour angle, which therefore may be had from one calculation. The hour-angle converted into time at the rate of 15° for one hour gives the apparent time from noon at the place of observation.

227. The latitude being known, the *variation of the compass* is easily found.

Previously to the discovery of the polarity of the magnetic needle, navigators had no means of ascertaining their course upon losing sight of land, but by the sun and stars, particularly the polar star. They therefore seldom dared to venture far from land, knowing that a short continuance of cloudy weather might occasion their destruction. On the discovery of the compass, an end was put to this difficulty. It must have been known at first that the needle did not point exactly north, but the deviation or *variation* was supposed everywhere the same. So slow was the progress of navigation, that nearly two centuries elapsed from the time that the polarity of the magnet

was well known in Europe, before it was discovered that in different places the variation was different. Columbus, in his first voyage, seems to have been the first who observed it. About a century later, the variation of the variation was discovered, that is, that the deviation from the north at a given place is variable. The variation at London, two centuries ago, was 11° 15′ east, and is now 19° 48′ west.

228. On these accounts it is obvious that the seaman must first ascertain the variation of the compass in the place in which he is, previously to his making use of it for his course : this he practises by a very simple astronomical observation : he notes, by the compass, the direction, called the bearing, of the sun when it rises or sets. If the bearing is measured from the east or west, it is called the *amplitude*. From the latitude of the place and the sun's declination, the azimuth at sunrise or sunset may be computed by the solution of a right-angled spherical triangle. For in the right-angled triangle formed by the sun's polar distance elevation of the pole and azimuth, cos. lat. : unity : : sin. dec. : cosin. azimuth. The difference of the amplitude observed and computed gives the variation.

Sometimes the sun's azimuth and altitude are observed : from the altitude, latitude, and declination, the azimuth may be computed, and thence the variation found : or knowing the latitude, sun's declination and time of day, the azimuth may be computed, and then compared with the azimuth observed.

229. Places not far distant have nearly the same variation, except near the poles.

It has been supposed that the variation of the needle, and latitude, would ascertain the position of a place, as well as its latitude and longitude ; and therefore that the variation of the needle would serve for finding the longitude. But the variation cannot be obtained with sufficient accuracy to apply it to this purpose. It seldom can be determined at sea nearer than a degree.

230. The next subject to be explained is the *method of finding the longitude at sea.*

The difference of the apparent times at two places, found by the difference of the angles between the hour circle passing through the sun, and the meridian at each place, at the same instant, is the difference of longitude, the whole equator being considered as divided into twenty-four hours, each of which corresponds to 15° of terrestrial longitude.

231. If then we have the time of day at any place, the situation of which is known, and compare it with the time at the place in which we are, we obtain by proportion the difference of longitude, allowing 15° for each hour. It is easy to find the time at the place we are in (Art. 226), and therefore the finding its longitude is reduced to find the time of day at some given place, as at Greenwich, from whence we, in these islands, reckon our longitude.

There are two methods of doing this at sea: by chronometers, as watches for this purpose are now usually called, and by making the motions of the celestial bodies serve instead of chronometers.

232. It is evident that did a watch or clock move continually at a uniform rate, it would afford us a ready means of finding the longitude: for if the chronometer, going mean time, were set to the time at Greenwich, it would continually point out the time at Greenwich, and therefore, by comparing that time with the mean time at the ship, we should at once have the difference of longitude between Greenwich and the ship. The apparent time at the ship can be found with all the accuracy necessary, and then applying the equation of time, the mean time will be obtained.

233. It became therefore an object of great importance to construct a machine, the uniform motion of which might be depended on for a length of time.

About the middle of the seventeenth century, Huygens and Hook made their celebrated improvements toward obtaining a regular movement in clocks and watches, the former by applying the pendulum to clocks,

and the latter by applying a spiral spring to the balance of watches.

Huygens himself proposed the pendulum clock for finding the longitude at sea, and quotes trials actually made; but it is obvious, on a variety of accounts, that a pendulum clock must be very unfit for a long voyage. Watches also, when made with the utmost care, were found to be by much too irregular in their rates of going to be depended on for a length time.

Under these circumstances an Act was passed in the reign of Queen Anne, in consequence of a petition from the merchants, for encouraging the discovery of a method of finding the longitude at sea within certain limits, for appointing a Board of Longitude, and for appropriating certain sums for encouraging attempts. It was understood that the most desirable method, on account of its easy practice, would be by chronometers. Mr. John Harrison early applied himself to the improvement of them, and during a long life was continually intent on that object. After many attempts which did his inventive genius the highest credit, and for which he received encouragement from the Board of Longitude, he at last completed a watch, which he considered perfect enough to entitle him to £20,000, the highest reward offered. Accordingly in the year 1761, a trial was made by sending the watch to the West Indies, and he was considered as entitled to £10,000, and the remainder was to be granted to him upon strictly complying with the terms of the Act. In the end, the whole, in consideration of his long and meritorious exertions, was granted to him.

The Act of Queen Anne only specified that to obtain the reward of £20,000, the error of longitude, in a voyage to the West Indies, should not exceed thirty miles. This, in time, is about an error of two minutes. Harrison's watch went within this limit: but it was soon found that the object of finding the longitude at sea, by time-keepers, was far from being attained. The construction of Harrison's watch was extremely difficult. It seems that not more than one or two have ever been made on his principles. He may be considered as having

led the way, and as having the credit of attempting the two principles of perfection which have for many years past been introduced in the construction of chronometers.

234. The two circumstances by which chronometers differ from common watches are, 1. The short time in which the mainspring acts upon the balance. This is accomplished by an escapement, called the detached escapement. The action of the mainspring is suspended during the greater part of the vibration of the balance, and therefore the isochronism of the balance spring is only slightly affected by the external impression of the main spring, through the intervention of the wheel work. 2. The contrivance for preventing the time of the vibration of the balance from being affected by heat or cold. The balance, instead of being an entire circle, as in common watches, is composed of two arcs (sometimes, but rarely, of three) to the end of each of which a small mass is attached: the external part of the arc is brass, and the internal part steel: these are soldered together, and from the different expansive powers of the two metals, by cold the arc becomes less curved, and by heat the contrary takes place. Thus the distance of the attached masses from the centre is always such as to preserve the isochronism. Chronometers well executed may be depended on to 1^s in a day.

It is evident, that, in long voyages, chronometers ought not to be trusted to, unless means of verifying them frequently offer. It is not necessary that a chronometer should not gain or lose time; all that is required is that it should gain or lose *uniformly*, and for this purpose it must be *rated* as frequently as opportunities will allow, in order that its *rate* of gaining or losing being known, allowance may be made for its gain or loss since the ship last left port. It is found that the rate of a chronometer at sea is not quite the same as on land, consequently several of them must be brought in order to act as checks on each other.

235. Of all the celestial bodies, the moon is to us far

the most convenient for the purpose of determining the longitude: its motion, as seen from the earth, being much quicker than that of the sun or any of the planets.

By the theory of the moon's motion, its place on the concave surface is known at any time; that is, knowing the time of the day at Greenwich, the place of the moon is known, and *vice versa* knowing the place of the moon, the time at Greenwich is known; so that if the lunar tables show that the moon, seen from the centre of the earth, will be 10° from a certain fixed star at 6 o'clock in the evening at Greenwich, and we make an observation at any distant place, and find that the moon's distance from the star, reduced by computation to what it would be seen from the centre of the earth, is 10°, we immediately conclude that it is 6 o'clock at Greenwich.

Thus the moon, with the brighter fixed stars near its path, may be considered as a chronometer, not made indeed by human hands, but perfect in its construction. It cannot, however, be easily used by us. The difficulty principally arises from slowness of the apparent motion of the moon on the concave surface, and therefore great nicety is required in measuring the angular distance of the moon from the fixed star. The intricacy of the lunar motions is also another source of difficulty.

But these inconveniences have now in a great measure been overcome by the improvements in instruments, and in the lunar theory; and navigators now use with much success this method.

236. It is briefly as follows:—

The observer measures the moon's distance from the sun, a planet or a bright star in the zodiac, specified in the Nautical Almanac, by means of a Hadley's sextant. This distance must be corrected for refraction, and reduced to the distance that would be observed from the centre of the earth, that is corrected for parallax. The lunar tables are formed to give the place of the moon, as would be seen from the centre of the earth in the meridian of Greenwich. For more readily computing the effects of parallax and refraction, another observer should, at the time of observing the distance, observe

the heights of the moon and star. These altitudes need not to be observed with great accuracy.

It being found, by a reference to the tables published in the Nautical Almanac, at what time the moon would be at this observed distance, so corrected, as seen at Greenwich, the time at Greenwich is known, and the difference of the local time and Greenwich time gives the longitude of the place..

To find the correct distance, or to clear, as it is termed, the observed distance from the effects of parallax and refraction, let Z be the zenith. The star is elevated in a vertical circle by refraction, and the moon is depressed by parallax and elevated by refraction also in a vertical circle, but as the former in her case is greater than the latter, she is on the whole depressed. We observe the zenith distances of the moon M, and star S, and the distance between them MS; from these three sides of a spherical triangle MZS, we calculate the difference of azimuths (the angle at Z) which is not altered by parallax or refraction. Now having by correction got the true zenith distances of the moon and the star and the vertical angle at Z thus calculated, we can compute the base which is their real distance.

237. The inconveniences of the lunar method of finding the longitude are—

1st. The great exactness requisite in observing the distance of the moon from the star or sun, as a small error in the distance makes a considerable error in the longitude. The moon moves at the rate of about a degree in two hours, or one minute of space in two minutes of time. Therefore, if we make an error of one minute in observing the angular distance, we make an error of two minutes in time, or 30 miles in longitude at the equator; since one degree at the equator, or 60 nautical miles, corresponds to 4 minutes of time. A single observation with the best sextants may be liable to an error of more than half a minute: but the accuracy of the result may be much increased by a mean of several observations taken to the east and west of the moon.

If the moon had moved round the earth in about

three days, the longitude would have been as easily found as the latitude. The first satellite of Jupiter enables the inhabitants of that planet to find their longitudes with as great accuracy as can be desired.

2nd. The imperfection of the lunar tables has also long been considered as an obstacle in this method. The improved tables of Mason were frequently erroneous by nearly one minute, which occasioned an error of thirty miles. But there is reason to suppose that the error of the new tables of Professor Hansen never exceed 10″, the average error is 3″, which is only equivalent to one mile and a half.

3rd. Another source of inconvenience is the length of the computation necessary in this method. Everything possible was done by the late Dr. Maskelyne for obviating this difficulty. He recommended the publication of the Nautical Almanac, which is now annually continued. In it the moon's distances from the sun and several zodiacal stars of the first and second magnitude are given for every three hours. Such plain rules also, for reducing the observed distance to the true, have been laid down, more particularly in publications directed by him, that the computation is very short, and merely mechanical, so that it cannot be mistaken by a person tolerably versed in arithmetic. This method of finding Greenwich time is most useful for correcting the chronometers.

By it the longitude will be generally known to less than ten miles, very often much nearer. This, although less accurate than the latitude, is an invaluable acquisition to the seaman: it gives him sufficient notice of his approach towards dangerous situations, or enables him to make for his port without sailing into the parallel of latitude, and then, in the seaman's phrase, running down the port on the parallel, as was done before this method was practised. In the last century navigators did not attempt to find their longitude at sea, unless by their reckoning, which was hardly ever to be depended on. The difficulties they experienced are easily conceived.

238. The present age must consider itself as principally indebted to the late Dr. Maskelyne, the Astronomer Royal, for the advantages which we derive from the lunar method of finding the longitude, and doubtless to him also posterity will acknowledge their great obligations. He, by his own experience, on his voyage to St. Helena, in 1761, first satisfactorily showed the practicability of this method. He strenuously recommended,[a] and then superintended, the publication of the Nautical Almanac, and of those tables, without the assistance of which this method would have been of little value to the seaman. To his observations is owing the perfection of the lunar tables formerly used; and he unremittingly assisted and encouraged every attempt to forward the discovery of the longitude at sea, whether by this method or by chronometers.[b]

239. It has been supposed that the eclipses of Jupiter's satellites might be of great use in finding the longitude at sea. Experience, however, has shown the contrary; it has been found impossible to manage a telescope on shipboard so as to observe the eclipses. All attempts to remedy this difficulty have hitherto failed, and on land it is difficult to be quite sure of the exact time of the beginning or the ending of the eclipse, the disappearance of the satellite taking place gradually.

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[a] *Vid.* Dr. Maskelyne's memorial, presented to the Commissioners of the Longitude, Feb. 9, 1765, printed in the Appendix to Mayer's Tables.

[b] The theory of the lunar method is very old: indeed, it is so obvious that it could scarcely have been overlooked in the infancy of astronomy; but the practice of it long seemed subject to insurmountable difficulties.

CHAPTER XIV.

GEOCENTRIC AND HELIOCENTRIC PLACES OF PLANETS—NODES AND INCLINATIONS OF THEIR ORBITS—MEAN MOTIONS AND PERIODIC TIMES—DISCOVERIES OF KEPLER—ELLIPTICAL MOTIONS OF PLANETS.

240. THE fixed stars, as has been noticed, appear in the same place with respect to the ecliptic from whatever part of the solar system they are seen, but not so the planets: their places, as seen from the sun and earth, are very different, and as their motions are performed about the sun, it is necessary to deduce from the observations made at the earth the observations that would be made by a spectator at the sun. By this we arrive at the true knowledge of their motions, and discover that their orbits are neither circular, nor their motions entirely equable about the sun, although a uniform circular motion will, as we have seen, in some measure solve the phænomena of their appearances.

241. It has before been shown how the distances and periodic times of the planets are found, on the hypothesis of their orbits being circular, and their motions uniform; it remains to show how the places of the nodes and inclinations of the orbits may be found nearly, before we proceed to more accurate investigations. For this it is necessary to find from the geocentric longitude and latitude (computed from the right ascension and declination observed), and the distance of the planet from the sun known nearly (Arts. 96 and 100), the heliocentric latitude and longitude.

242. Let S and E (Fig. 41) be the sun and earth, P the planet, O its place, projected perpendicularly on the plane of the ecliptic, SA the direction of Aries, and

EH parallel to SA. Then OEH and OEP are the geocentric longitude and latitude of the planet, and OSA and PSO are the heliocentric longitude and latitude. From the right ascension and declination observed, and the right ascension and declination of the sun, we can compute, by spherical trigonometry, the planet's angular distance from the sun, or the angle SEP. For we have then the angle at the pole between great circles drawn from it to the sun and the planet, and the polar distance of each. Therefore in the triangle SEP we know SP, SE, and the angle SEP; from thence we can deduce PE, and thence OE, because OE = PE × cos OEP (geocent. lat.). Hence in the triangle SOE, ES, OE, and angle SEO (diff. long. of planet and sun) are known, and so we can compute OSE. Whence, because ESA = earth's longitude seen from sun = sun's longitude + 180°. we obtain OSA, the heliocentric longitude. Also because PS × sin PSO = OP = EP × sin OEP, we have sin. hel. lat : sin geo. lat. : : EP : PS, and thus the heliocentric latitude is known.

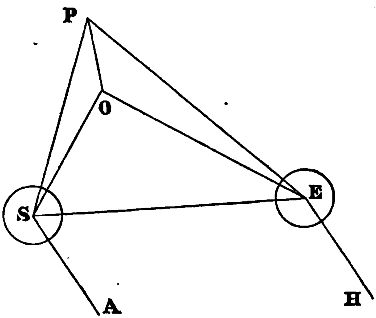

Fig. 41.

243. From two heliocentric longitudes and latitudes, the place of the node and inclination of the orbit may be found. Let l and l' be two heliocentric longitudes, λ and λ' the heliocentric latitudes, x the longitude of the ascending node, and ϕ the inclination of the planet's orbit to the ecliptic. Then by spherical trigonometry it is easy to see that we have two right-angled triangles with a common angle ϕ, and $l-x$, λ, $l'-x$, and λ' the sides, which give $\sin(l-x)=\tan\lambda.\cot\phi$ (α); $\sin(l'-x)=\tan\lambda'.\cot\phi\ldots(\beta)$; ∴ expanding, and dividing (α)

by (β), $\sin l \cos x - \cos l \sin x = \frac{\tan \lambda}{\tan \lambda'}\{\sin l' \cos x - \cos l' \sin x\}$, dividing by $\cos x$, we have a simple equation to solve for $\tan x$, and we easily obtain $\tan x = \frac{\sin l \tan \lambda' - \sin l' \tan \lambda}{\cos l \tan \lambda' - \cos l' \tan \lambda}$. This being found, we have

$\cot \phi$ (inclin. of orbit) $= \frac{(\sin l - x)}{\tan \lambda}$.

The best observations for ascertaining the place of the node are those made when the planet is near its node on each side: the best for ascertaining the inclination are when the planet is farthest from the ecliptic.

The above is applicable to finding accurately the place of the node and inclination of the orbit, provided we had the planet's distance from the sun, at each observation, accurately. How these may be found will appear farther on. Therefore thus far it has only been shown how the distances, periodic times, places of the nodes, inclinations of the orbits, may be nearly found.

244. Among the most valuable observations for determining the elements of a planet's orbit, are those made when a superior planet is in or near opposition to the sun, for then the heliocentric and geocentric longitudes are the same. And a number of oppositions being observed, the planet's motion in longitude, as would be observed from the sun, will be known. The inferior planets also, when in superior conjunction, have the same geocentric and heliocentric longitudes: when in inferior conjunction they differ by 180°; but the inferior planets can seldom be observed in superior conjunction, or in inferior conjunction, except when they pass over, which they rarely do, the sun's disc. Therefore we cannot so readily ascertain by simple observation the motions of the inferior planets, seen from the sun, as we can those of the superior.

245. The principal element for determining the place of a planet is the mean angular velocity about the sun called the *mean motion.* The periodic time is considered as invariable; but neither the real motion in

its orbit, nor its angular motion about the sun is equable. The periodic time, being constant, may be taken as the measure of its mean motion; or rather the mean angle described in any given time, as twenty-four hours (deduced by proportion, from 360° being described in the periodic time).

If the planet's place in its orbit, as seen from the sun, at any time, be known, its place at that time will be had within a few degrees or less, by adding its mean motion, in the interval, to the former place: this is to be corrected according to the deviation of the trne motion from the mean place.

To obtain *accurately* the periodic time of a planet. Find the interval elapsed between two oppositions separated by a long interval, when the planet was nearly in the same part of the zodiac. From the periodic time known nearly, it may be found when the planet has the same heliocentric longitude as at the first observation. Hence the time of a complete number of revolutions will be known, and thence the time of one revolution. The greater the interval of time between the two oppositions, the more accurately the periodic time will be obtained, because the errors of observation will be divided among a great number of periods; therefore, by using very ancient observations, much precision may be obtained.

245. The planet Saturn was observed in the year 228 B. C., March 2 (according to our reckoning of time) to be near the star γ Virginis, and at the same time was nearly in opposition to the sun. The same planet was observed in opposition to the sun, and having nearly the same longitude, in February, 1714.

Whence it was found that 1943 common years, 118 days, 21 hours, and 15 minutes had elapsed while the planet made 66 revolutions. It being readily discovered that the time of a revolution was $29\frac{1}{2}$ years nearly, it was easily ascertained that exactly 66 revolutions had been completed in the above interval, and therefore it follows that $29^{y}\ 162^{d}\ 4^{h}\ 19^{m}$ is the time of one revolution, which gives $2'\ 0''$, 58 for the mean motion in 24 hours. The

above time of revolution is with respect to the equinoctial points, and, as the equinoctial points recede, the time of a complete revolution in the orbit will be had by finding the precession of the equinoxes in longitude in the above time of revolution, and thence computing, by proportion, the time the planet takes to go over the arc of longitude equal to the precession. In this way the time of a complete revolution is found to be 29^y 174^d 11^h 29^m: this is called a *sidereal revolution*, because it is the time elapsed between two successive returns of the planet to the same fixed star, when seen from the sun. The time of revolution with respect to the equinoxes, the same as the time of revolution with respect to the tropics, is called the *tropical revolution*.

In the same manner ancient observations have been used for the other planets. Ptolemy has recorded several oppositions of Jupiter and Mars observed by him in the second century. From these Cassini computed, by the help of modern observations, the periodic times with much exactness. Ancient observations have also been used for Venus. Mercury, before the invention of telescopes, could not be seen, when near either inferior or superior conjunction, and therefore for this planet modern observations only have been used: however, its transits over the sun's disc have enabled us to obtain the periodic time with sufficient accuracy.

246. The *exact* periodic time of the earth is readily found by a comparison of two distant equinoxes; the time of the equinox is known by observing the sun's declination before and after the equinox, and thence the time when the sun had no declination may be computed by proportion. Comparisons of good observations, separated by a long interval, give the time of returning to the same equinox, or the length of a *tropical* year = 365^d 5^h 48^m 48^s, and as the recession of the equinoctial points is $50\frac{1}{4}''$ in a year, the sun will appear to move over this space in 20^m $23\frac{1}{2}^s$. Hence the periodic time of the earth or a *sidereal* year = 365^d 6^s 9^m 11^h.

247. The ancient observations of Jupiter and Saturn, compared with the modern ones, give the periodic time

of the former greater, and that of Saturn less, than what are found by a comparison of the modern observations. The cause of this is satisfactorily explained by the mutual attraction of those bodies to each other, and the quantity of variation has been computed by the help of physical astronomy.

The tropical year is less now than in the time of Hipparchus, according to the determination of Laplace, by about 10^s.

248. KEPLER'S LAWS.—The next inquiry is the deviation of a planet's motion from equal motion, for which the knowledge of the form of the orbit, and law of motion in that orbit, is necessary. This brings us to the discoveries of Kepler, who first ascertained, from the observations of Tycho Brahe—1° that the *planets move in ellipses about the sun, which is in one of the foci;* 2° that the law of the motion of each planet is such, that it *describes about the sun equal areas in equal times;* and 3° that *the squares of the periodic times are as the cubes of the greater axes of their orbits.* Kepler, to whom we owe these important discoveries, was born in 1571, and distinguished himself early in the seventeenth century. Naturally possessed of a most ardent desire of fame, it was fortunate for the progress of astronomy that he applied himself to this science. He had the advantage of referring to the numerous and celebrated observations of Tycho Brahe; who having, with unwearied exertions, constructed instruments far better than had ever been made, used them with equal assiduity in forming a connected series of most valuable observations. Tycho Brahe observed in the Island of Huine, near Copenhagen; from whence, in consequence of most unmerited treatment, he was obliged to retire to Prague, whither Kepler, at his persuasion, came to reside.

249. Kepler first applied himself to investigate the orbit of Mars,[a] the motions of which planet appeared more irregular than those of any other, except Mercury,

[a] This was merely accidental. Vid. Kepler De Motibus Stellæ Martis, p. 53.

which, from being seldom seen, had been little attended to. He has left us the result of his inquiries in his work, "De Motibus Stellæ Martis," which will always deserve to be studied as a record of industry and ingenuity. It will not be convenient to enter here into many particulars of his labours. One of the most remarkable is, his long adherence to the hypothesis that the orbits of all the planets must be circular, because a circle is the most perfect figure. The planet was supposed to move in a circle describing equal angles about a point (punctum æquans) at a certain distance from the sun. In this he was sanctioned by all who had gone before him, and it was not till having in vain spent nearly five years in attempting to accommodate this hypothesis to the observations that he could persuade himself to reject it. "Primus[b] meus error fuit viam planetæ perfectum esse circulum; tanto nocentior temporis fur, quanto erat ab authoritate omnium Philosophorum instructior et metaphysicæ in specie convenientior." He afterwards proceeded by a method in which all conjecture was laid aside. From the numerous observations of Tycho Brahe, that had been continued upwards of twenty years, he obtained many distances of Mars from the sun, and the angles at the sun contained by these distances, and at last discovered that the curve passing through the extremities of these distances was an ellipse; in this manner arriving at a conclusion which he considered as fully repaying him for his trouble. His attempts, his repeated disappointments, all of which he has ingeniously recorded; his ready invention in surmounting difficulties; his perseverance at last crowned with success; remain as highly useful examples to show the value of genius and industry united. His adherence to the circular hypothesis, which was principally supported by its antiquity, affords a useful illustration of the inconveniences that may arise from not taking experiment and observation for our guides; and by his ultimate success he may be said

[b] De Motibus Stellæ Martis, cap. 40, p. 192.

to have given an illustrious example of that method of philosophising which a few years afterwards was so strenuously recommended by Lord Bacon.

250. Kepler's method, by which he at last obtained the orbit of Mars, will serve as a plain example of the manner of finding the orbit of a planet, and therefore may be considered as proper for an elementary work, although the present advanced state of astronomy furnishes others more convenient, but not so simple.

He considered the orbit of the earth as circular, the sun being at a small distance from the centre, which the observations of Tycho were not sufficiently accurate to contradict, the orbit of the earth deviating so little from a circle. Thus he was enabled to ascertain with sufficient precision the relative distances of the earth from the sun at different times, and the angles described about the sun; having discovered that the point round which motion was equable was not, as astronomers at that time supposed, in the centre of the circle, but in the continuation of the line joining the sun and centre, and equally distant from the centre as the sun.[a]

Let T and E (Fig. 42) be two places of the earth, when Mars is in the same place of its orbit—(these times are known from knowing the periodic time of Mars)—P Mars, and M its projection on the plane of the ecliptic; S the sun. The angles MTS and MES are known from observations; TS, SE, and the angle TSE from knowing the orbit and motion of the earth. In the triangle TSE we can find STE and TES and TE. From these angles we find MTE and MET, and thence by help of TE we compute MT. Knowing MT, TS, and the included angle, we find MS by the proportion MT : MS : : cot. PTM (geo. lat.) : cot. PSM (hel. lat.) thus we obtain the heliocentric latitude. Then cos. PSM (hel. lat.) : rad. : : SM : PS.

251. By the numerous observations of Tycho Brahe,

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[a] The ancient astronomers had supposed this to be so with respect to the planets, but the hypothesis had been rejected by Copernicus. It is only nearly true in the orbits that are of small eccentricity.

Kepler was enabled to verify the same distance from several pairs of observations, and also to find many different distances, and the angles at the sun contained by these distances. In this manner he also found the greatest and least distances. Supposing the orbit circular, he had from these the diameter of the circle, and he could deduce any other distance at pleasure; by which means he compared the distances computed on this hypothesis with the distances computed from observation, and found that the distances in the circle were always greater than the observed distances. Hence he was assured that the orbit was not circular, but oval. He was at last led to try an ellipse, having the sun in one of the foci: this he found to answer by a comparison of a great number of observations of Mars. He concluded the same to be true for all the planets, and soon ascertained that *each described equal areas in equal times round the sun.*

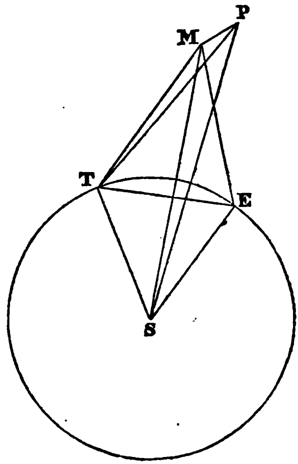

Fig. 42.

252. The last discovery of Kepler was, that the *squares of the periodic times are as the cubes of the greater axes of the ellipses.* This discovery was made many years after the two former: he conceived there must be some relation between the motions of the respective planets, which led him to search for that relation, and the above law was the result, which seems to have given him as much pleasure as any of his discoveries. We now know that this remarkable proportion is a simple result from the principle of universal attraction

which pervades all bodies. How great must have been the satisfaction of Newton, who first established the existence of universal gravity, and by the application of mathematical principles showed that the three famous discoveries of Kepler were necessary consequences of that universal property of bodies.

253. It will not be convenient here to enter into a further detail of the methods by which all the particulars of the elliptical motions of the planets have since been established. They may be found in the copious astronomical treatises of Lalande, Professor Vince, Delambre, and others.

The computations made from the elements of the elliptical motions agree so precisely with observation, that not a shadow of doubt can remain, that the planetary motions are performed according to the above laws; and all that may be thought necessary here is to show briefly, how the geocentric place of a planet may be computed from the elements of its motion in an elliptic orbit about the sun, and so compared with the same given by observation.

254. When a planet is at its greatest and least distances from the sun, it is said to be in *Aphelion* and *Perihelion*. The ratio of the distance of the sun from the centre of the ellipse to the semiaxis major, is called the *eccentricity of the orbit*. If the angular motion of the planet about the sun were uniform, the angle described by the planet in any interval of time, after leaving Aphelion, might be found by simple proportion, from knowing the periodic time in which it describes 360°: but as the angular motion is slower near Aphelion, and faster near Perihelion, to preserve the equable description of areas, the true place will be behind the mean place in going from Aphelion to Perihelion; and from Perihelion to Aphelion, the true place will be before the mean place (p. 171). The angle at the sun contained between the true and mean place is called the *equation of the centre*. The angle between the Aphelion and mean place is called the *mean anomaly*,

and the angle between the true place and Aphelion the *true anomaly.*

255. The tables give the mean place of the planet in its orbit at some given time, called the epoch; from thence the mean place at any other time may be found either by the tables, or by proportion: if from this the place of Aphelion be subtracted, the mean anomaly of the planet is obtained, and from thence the true place is to be found. The numerous calculations, now requisite in astronomy, make it necessary that all the aid possible should be derived from tables. Accordingly the tables give the mean motion about the sun for years, days, hours, &c., the place of the Aphelion,[a] and the equation of the centre and distance from the sun, for different degrees of mean anomaly. Thus we obtain the true place of the planet as seen from the sun, and its distance from the sun. The difference between the place in its orbit and the place of the node gives its distance from the node; whence, from knowing the inclination, we can compute its angular distance on the ecliptic from its node, and also its angular distance from the ecliptic, and thus find its heliocentric longitude and latitude. Hence, knowing the earth's distance from the sun, and its place, as seen from the sun, we can compute, by the converse of the method in Art. 242, the geocentric latitude and longitude.

The best tables of the motions of the planets contain the corrections to be applied on account of the mutual attraction of the bodies of the system, by which their motions are disturbed, and by which also their nodes and Aphelia slowly change their places. In general three complete observations of a planet determine the orbit in which it is moving. This may be seen easily from the

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[a] The latest French tables reckon the anomaly from Perihelion, instead of Aphelion, as has been usual hitherto. This was done to make the mode of reckoning similar to that for comets, the motions of which are necessarily estimated from Perihelion; and the intention seems to be, that in future the anomaly of the planets should be computed in the same manner.

geometry of the conic sections; for if we have three focal radii vectores of an ellipse, FA, FB, FC, and the angles between them, AFB, BFC, we can determine the curve completely as follows:—Assume the directrix (which is to be determined) to be known, and let perpendiculars AR, BS, CT be let fall on it from A, B, and C, then join BA, and produce it to meet the directrix in X, we have BX : AX : : BS : AR : : FB : FA; therefore X is known. In the same manner, if CB be produced to meet the directrix in Y, we have CY : BY : : CT : BL : : FC : FB; consequently Y is known, and therefore XY the directrix is given in position. Let fall a perpendicular FZ from the focus on its direction, and cut it internally at G, and externally at H, in the ratio of FA : AR, and G and H will be the extremities of the greater axis.

256. RELATIONS BETWEEN THE MEAN ANOMALY AND THE ECCENTRIC; AND BETWEEN THE ECCENTRIC AND THE TRUE.—*The true place of the planet in the ellipse, or the*

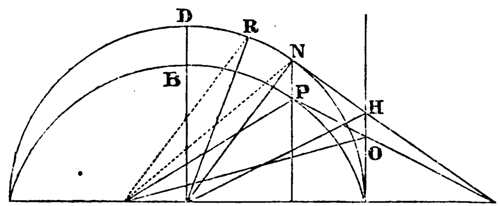

Fig. 43.

true anomaly (v) *may be deduced* from the mean anomaly (m) by means of another angle called the *Eccentric Anomaly* (u), which is the angle MCN at the centre of the ellipse, between the greater axis and the line CN, drawn from the centre to the place N, where a perpendicular to that axis, PM, when produced, meets the semicircle described on the greater axis of the ellipse. Let FN join the focus F with the point N, and ACR be the mean anomaly. Let t be the time from Aphelion, and T the time of describing the semiellipse, it is evident that the

sector ACR : area of semicircle : : mean anomaly : 180° : : t : T : : area of elliptic segment AFP : area of semi-ellipse (by Kepler's second law) : : area of the figure AFN : area of semicircle on the greater axis; ∴ the sector ACR = figure AFN = the sector ACN + the triangle FCN, consequently $\frac{1}{2}AC^2 \times$ angle ACR $= \frac{1}{2}AC^2 \times$ angle ACN $+ \frac{1}{2}FC \times CN$ sin MCN, but FC = CA × eccentricity (e), and CN = CA, consequently dividing by $\frac{1}{2}AC^2$; ACR = ACN + e sin MCN. As the anomalies are reckoned from perihelion, we must increase these angles each by 180°, and consequently we have the relation[a]

$$m = u - e \sin u.$$

In order to find the relation between the eccentric and the true anomaly, draw the tangents NL, PL, and let them cut at O and H the common tangent AH drawn at A to the semicircle, and the semiellipse; join CH and FO; by the properties of the ellipse NCA is bisected by CH, and PFA is bisected by FO; consequently $\frac{HA}{AC} = \tan \frac{1}{2} u$. and $\frac{OA}{AF} = \tan \frac{1}{2} v$ (counting from Aphelion A), and consequently, $\tan \frac{1}{2} u : \tan \frac{1}{2} v$

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[a] We have seen above that the area AFN = the sector ACR, taking away the common part, ACN, the triangle FCN = the sector NCR (which may be considered as a triangle since RN is small), consequently CN and FR are nearly parallel, and CFR = the eccentric anomaly nearly. Therefore in the triangle FCR, FC + CR : FC − CR : : tan $\frac{1}{2}$ (CFR + FRC) : tan $\frac{1}{2}$ (CFR − FRC) : : tan $\frac{1}{2}$ mean anomaly : : tan $\frac{1}{2}\,\delta$. Let CFR $= u'$ and $\delta = 2u' - m'$ consequently the mean anomaly $= 2u' + \delta$. Let the difference between the strict eccentric anomaly (u), and the approximate (u'), be x, then $u = u' + x$; but the mean anomaly $(m) = u - e \sin u = (u' + x) - e \sin (u' + x) = u + x - e \sin u' - e\, x \cos u'$ (since x is very small); therefore, if $u' - e \sin u' = m'$; $m = m' + x\,(1 - e \cos u')$, consequently $x = \dfrac{m - m}{1 - e \cos u'} = \dfrac{m - m'}{2 \sin^2 \frac{\phi}{2}}$ where $e \cos u' = \cos \phi$. The eccentric anomaly thus found $(u + x)$ will be sufficiently exact to give the true anomaly to less than 1″ for all the planets.

in a ratio compounded of the ratios of HA : OA, and of AF : AC; but HA : OA : : NM : PM : : DC : BC (by the properties of the ellipse), consequently multiplying these ratios we obtain the following relation : $\tan \frac{1}{2} u : \tan \frac{1}{2} v : : \mathrm{AF} : \mathrm{BC} : : \mathrm{AC} + \mathrm{CF} : \sqrt{\mathrm{AC}^2 - \mathrm{CF}^2} : : \sqrt{\mathrm{AC} + \mathrm{CF}} : \sqrt{\mathrm{AC} - \mathrm{CF}}$,

$$\tan \tfrac{1}{2} u = \sqrt{\frac{1+e}{1-e}} \tan \tfrac{1}{2} v.$$

257. The problem for finding the true from the mean anomaly, or, which comes to the same, to divide the area of the semicircle, by a line drawn from a point in the diameter, in a given ratio, has long been celebrated; and known by the name of *Kepler's problem;* he first endeavoured to solve it in consequence of his discovery that a planet describes equal areas in equal times about the sun. No exact solution can be given; it must be done either by continued approximation, or by help of a series.

258. Astronomers were not long in adopting Kepler's discovery of the elliptical motions of the planets, but they long hesitated in adopting the equable description of areas, in consequence of the difficulty it involved of finding the true from the mean place. They instead thereof had recourse to such hypotheses for the law of motion as would afford them easy rules for finding the true from the mean place, and at the same time would give the computed place nearly within the limits of the

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[a] It has generally gone by the name of Ward's Solution; yet he did not claim it as his own, but acknowledged himself indebted to Bouilliald for the hint that led him to it. The fact is, that Kepler himself was not ignorant of it as an approximation, but rejected it as not sufficiently accurate. It is easy to prove, geometrically, that if we take two consecutive points P, P' on an ellipse, and join each of them with the two foci F, F' that the small perpendicular from P on FP' is equal to the small perpendicular from P' on F'P, consequently the angular velocity round F : that round F' : : F' P : FP; but the angular velocity round F varies inversely as the square of FP (by Kepler's second law), and hence that round F varies inversely as FP·F'P, that is as the inverse square of the semiconjugate diameter.

errors of observation. One of the most celebrated was that of Seth Ward,[a] known by the name of the *Simple Elliptic Hypothesis:* its value was derived, not from its accuracy, but from the elegance of the analogy used. He supposed the motion equable about the focus in which the sun was not; and from thence it easily follows that the Aphelion dist. : Perihelion dist. : : tan $\frac{1}{2}$, mean anomaly : tan. $\frac{1}{2}$ true anom.; for, if P be a point on an ellipse, and F and F′ the foci; join the focal distances FP, F′P; and produce F′P to Q, so that PQ = PF and join QF. In the triangle QFF′, QF′ + FF′ : QF′ − FF′ : : tan $\frac{1}{2}$ (F′FQ + FQF′) : tan $\frac{1}{2}$ (F′FQ − FQF′); that is, 2 Aphelion dist. : 2 Perihelion dist. : : tan $\frac{1}{2}$ mean anom. : tan $\frac{1}{2}$ true anom. The anomaly thus found may sometimes differ in the orbit of the planet Mercury 33′ from the truth, and in that of Mars 7′. As no satisfactory reason could be assigned for Kepler's law, any other law that appeared to show with equal accuracy the motions of the planets about the sun had an equal claim to attention. This occasioned the invention of several different hypotheses before the time of Sir Isaac Newton: but his discoveries having fully established the Keplerian law, they were soon laid aside.

The first approximation above given for the eccentric anomaly may occasion an error of 5′ in the anomaly of Mercury, of 20″ in that of Mars, &c.

The eccentricity of the earth's orbit being the difference of the Aphelion and Perihelion distances divided by their sum is equal to the difference between the sun's greatest and least apparent diameters, divided by their sum, that is 32′ 34″ − 31′ 29″, divided by 64′ 3″, or $e = 0.01688$, (0.01677 from the most accurate observations) or $\frac{1}{60}$ very nearly.[a]

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[a] Adopting Ward's hypothesis the angle FPF′ would approximately represent the equation of the centre, which is a maximum at the end of the axes minor where the sine of its half = the eccentricity, in the case of the earth $\frac{1}{60}$ nearly, this would correspond to sin 58′, nearly, twice which would give for the equation of time arising from this cause $7\frac{3}{4}$ minutes (page 171).

It is shown that the earth describes an ellipse round the sun, from the fact that its distance, which is inversely as the sun's diameter, is proportional to $1 + e$ cos. true anomaly.

259. The following table exhibits the elliptic elements of the orbits of the principal planets.

	Merc.	Ven.	Earth.	Mars.	Jup.	Sat.	Uranus	Nep.
Eccentricity of the orbit, the mean distance 1000.	205⅓	7	17	93	48	56	46½	9
Places of the Aphelion seen from the sun.	s o 8 14	s o 10 8½	s o 9 9½	s o 5 2	s o 6 11	s o 8 29	s o 11 17	s o 7 17
Mean motion in 24 hours as seen from sun.	o ′ ″ 4 5 3	o ′ ″ 1 36 8	″ 59 8	″ 31 27	′ ′ 4 59	′ [illegible] 0.6	′ ″ 0 42 4	′ ″ 0 21.6
Greatest equation of centre or deviation from mean place.	o ′ 23 40	o ′ 0 47	o ′ 1 55	o ′ 10 40	o ′ 5 30	o ′ 6 27	o ′ 5 21	

Vesta .	The eccentricity of the orbit	90
	Place of Aphelion	2ˢ 9° 20′
	Mean motion in 24 hours .	16′ 18″
Juno .	Eccentricity of the orbit .	257
	Place of Aphelion	8ˢ 22° 49′
	Mean motion in 24 hours .	13′ 35″
Ceres .	The eccentricity of the orbit	80
	Place of Aphelion	10ˢ 26° 9′
	Mean motion in 24 hours .	12′ 50″,7
Pallas .	Eccentricity of the orbit .	240
	Place of Aphelion	10ˢ1° 7′
	Mean motion in 24 hours .	12′ 50″,9
Astræa	Eccentricity of the orbit .	190
	Mean motion in 24 hours .	14′ 18″

Three of the Asteroids have eccentricities exceeding $\frac{3}{10}$, Polyhymnia, .338; Euridice, .306; Atalanta, .3007. Faye's comet has an eccentricity of .556.

CHAPTER XV.

THE MOON—ITS PHYSICAL CONSTITUTION—ITS MOTIONS—MOTIONS OF SATELLITES—COMETS.

260. Physical Constitution of the Moon.—The great improvement made in the construction of telescopes in recent times has made us acquainted with the details of the lunar surface to an extent to which we could not formerly approach. We have already noticed (Art. 134) the existence of lunar mountains, as proved by the bright spots which appear on the dark part, and also by the shadows which are specially conspicuous near the circle of light and darkness, when the altitude of the sun is low. These are seen only in the phases; none are visible when the moon is full, because in that position the visual ray coincides with the direction of the rays from the sun. A peculiarity of the lunar mountains is that they are for the greater part of a more or less regularly circular shape. The smaller formations of this circular class are called craters; the larger ones, varying from 10 to 50 miles in diameter, are called ring mountains. These circular mountains bear a striking resemblance to volcanoes on our earth; but they are of vastly larger dimensions, inasmuch as the largest terrestrial craters, such as that of Kilauea, in the Sandwich Islands, which is three miles in diameter, and 1000 feet deep, would be equal to only the smallest of these formations on the moon. We have already noticed (Art. 134) that Sir W. Herschel supposed that some bright spots indicated the existence of active volcanoes on the moon. However, Beer and Mädler, who have published the most complete description of the lunar

surface, in their work entitled "Der Mond," state that there is no trace of any present volcanic action on its surface. The most remarkable of the ring mountains is called Tycho, near the moon's southern limb. Their outlines are generally very regular, and nearly circular; their inner declivity is always very steep, and the enclosed area concave, and lower than the region surrounding the enclosure. In the centre there is often a central peak or cone, thus presenting the general character of a volcanic centre. The largest of these circular formations, up to 120 miles in diameter, are called bulwark planes. They are less regular, and the ring of mountain ridges is often intersected by large ravines. The enclosed area, in their case, is generally a plain, on which small mountains and peaks are scattered. Although this *circular* formation of the lunar mountains is prevalent, there are, nevertheless, some mountain ranges which resemble those on the earth, as the Alps, &c., though they are entirely destitute of the large longitudinal valleys of our mountains.

Another striking feature of the lunar surface is the variety in the intensity of light which it reflects. There are large uniform patches of a greyish tint, which were supposed by early observers to be large collections of water, and are still designated by such names as Mare Nubium, Mare Serenitatis, &c.; but from what has been already stated (Art. 133), they cannot be water; they rather show irregularities and undulations of permanent forms. These are very extensive, and occupy about two-thirds of the visible hemisphere of the moon. Another remarkable feature is the existence of streaks of light, which radiate from the borders of some of the ring mountains, such as Tycho, Kepler, Copernicus, &c.; light from the former extending to a distance of several hundred miles. These are specially conspicuous at the time of full moon. Some have supposed them to arise from lava, melted and afterwards vitrified, filling enormous fissures in the moon's surface, caused by violent volcanic action.

The chart of Beer and Mädler, of 37 inches diameter,

exhibits the most complete representation of the lunar surface. They have measured the height of a large number of lunar mountains by the second method indicated at the end of Art. 135; and in their work already mentioned have given a list of heights, varying up to 22,823 feet. These heights are, however, not to be understood in the same manner as the height of terrestrial mountains, which are measured above the level of the sea. Lunar mountains can be referred, by this method of measurement, only to the surrounding plains, of whose elevations nothing is known. They are, consequently, simply *differences* of elevation.

201. The most recent experiments show that little, if any, heat exists in the light reflected from the moon's surface, and that the moon absorbs nearly all the heating rays which lie at the red end of the solar spectrum, while she reflects largely the violet or blue rays, and with them the chemical rays which lie beyond the other end of the spectrum.

The Earl of Rosse is at present engaged in researches as to the radiation of heat from the moon. His observations go to prove that the moon's heat can be detected with certainty at any time between the first and last quarter, and that it varies with the moon's phase; that its increase is proportional to that of the moonlight, and that a large portion of the rays of high refrangibility, which reach the moon from the sun, do not at once leave the moon's surface, but are first absorbed, raise the temperature of the surface, and afterwards leave it as heat-rays of low refrangibility. His experiments make the sun's total heat = 82,600 times the moon's total heat. They also tend to show that the heating power of the moon's rays diminishes with her altitude only about one-third as fast as the intensity of the solar chemical rays.[a] It is popularly

[a] Proceedings of the Royal Society, vol. xix., p. 9.

believed that the changes of the moon influence the weather. There is no reason to think, from the results of long-continued observation, that such is the case. Sir John Herschel notices the fact—also shown by the observations of M. Arago—that the skies are generally more cloudless when the moon is full, than at new moon. This he explained by the absorption of whatever heat is reflected from the full moon by the upper regions of the atmosphere, which thus dispels the clouds at that time. Professor Loomis, however, states that, from a comparison of seven years' observations at Greenwich, he has found that at full moon the average cloudiness of the sky is precisely the same as at new moon. We have seen already (Art. 142) that no appreciable atmosphere surrounds the moon, so that, without either air or water, and consequently vegetation, its circumstances must differ widely from those of the earth.

262. Motions of the Moon.—The satellites, as well as the planets, are found to revolve in elliptic orbits round their respective primary planets, having the same law of periodic times; but considerable deviations from the equable description of areas take place, in consequence of the disturbing force of the sun on the satellites, and of the satellites on each other.

The moon being a solitary satellite, we cannot apply the law of the periodic time to it. But its orbit is nearly an ellipse, and it nearly describes areas proportional to the times, the deviation from which arises from the disturbing force of the sun. This ellipse, however, does not retain the same position; that is, its points of greatest and least distance, called is *apogee* and *perigee*, do not retain the same position, but move according to the order of the signs, completing a revolution in about nine years.

The laws of the principal irregularities[a] of the moon were discovered long before the cause of them.

[a] The correction for these irregularities (improperly so called) are styled *equations*.

263. The greatest difference between the true and mean place of the moon, arising from its elliptic motion, or the greatest equation of the centre, is 6° 18′, and this is the most considerable deviation from its mean place. But besides the quick motion of the apogee, completing a revolution in nine years, the eccentricity of the ellipse is also variable: hence the motions of the moon appear so irregular that it would have been almost impossible to have developed the elliptic motion from the phenomena; and therefore, without a knowledge of the form of the planetary orbits, it is hardly to be supposed that an ellipse could have been applied for explaining the motions of the moon, although at first sight the superior advantage of being in the centre of the orbit might lead us to suppose that the laws of its motions would be more easily known.

264. The periodic time of the moon may be ascertained with great exactness from the comparison of ancient eclipses with modern observations. At an eclipse of the moon, the moon being in opposition to the sun, its place is known from the sun's place, which can, back to the remotest antiquity, be computed with precision. Three eclipses of the moon, observed at Babylon in the year 720 and 719 B.C., are the oldest observations recorded with sufficient exactness. By a comparison of these with modern observations, the periodic time of the moon is found to be $27^d\ 7^h\ 43^m\ 11\frac{1}{2}^s$, not differing half a second from the result obtained by recent observations. Yet we cannot use those ancient observations for determining the mean motion at the present time; for by a comparison of the above-mentioned eclipses with eclipses observed by the Arabians in the 8th and 9th centuries, and of the latter with the modern observations it is well ascertained that the motion of the moon is now accelerated. This was first discovered by Dr. Halley, and since his time has been perfectly established by more minute computations. For a considerable time the cause remained

unexplained, till Laplace showed it to be a variation of a very long period, arising from the disturbance of the planets in changing the eccentricity of the earth's orbit. He has computed its quantity, which closely agrees with that deduced from observation. In the year 1853, however, Professor Adams pointed out a deficiency in Laplace's method of deducing the effect which the secular change of the eccentricity of the earth's orbit produces on the mean motion of the moon, and found that the true acceleration due to this cause is only about one-half of what Laplace had calculated. This result was fully corroborated by the researches of Delaunay and Cayley. The moon's secular motion, the motion in a century, is now $7\frac{1}{2}'$ greater than it was at the time the above-mentioned eclipses were observed at Babylon.

265. The two principal corrections of the mean place of the moon, beside that of the equation of the centre, are called the *evection* and *variation*. The evection depends upon the change of the eccentricity of the moon's orbit, and sometimes amounts to $1° \ 20' \ 30''$. This was discovered by Ptolemy (A. D. 140). The variation which was discovered by Tycho Brahe (A. D. 1580) depends upon the angular distance of the moon from the sun, and amounts, when greatest, to $39' \ 30''$. He also discovered another irregularity, called the *annual equation*, which diminishes the longitude by $11' \ 9'' \times$ sin. (*sun's anomaly*). The other corrections arise only to a few minutes. But the number of corrections or equations used at present in computing the longitude alone of the moon are thirty-two, and in computing the latitude twelve.

266. It was before mentioned that the nodes of the lunar orbit move retrograde, completing a revolution in eighteen years and a half. This motion is not uniform, as was first noticed by Tycho Brahe. He also found that the inclination of the orbit remains nearly the same, but not exactly[a]. The motion of the apogee is

[a] The *mean* value of the inclination is $5° \ 8' \ 56''$. It varies from $4° \ 57' \ 2'$ to $5° \ 20' \ 6''$. The nodes make a complete revolution in $18^y \ 218^d \ 21^h \ 22^m \ 46^s$.

subject to considerable irregularities: its true place sometimes differs $12\frac{1}{2}°$ from its mean place. This was known to the Arabian astronomers, but seems to have been first accurately stated by Horrox (A. D. 1639), an astronomer of extraordinary powers whose discoveries have been already noticed. He showed the law of its change, and gave a construction for determining its quantity which was adopted by Newton.

267. On all these accounts the computation of the exact plaee of the moon from theory is very difficult, and the formation of proper tables is one of the greatest intricacies in this science.

No small degree of credit is due to the industry of those who, by observation alone, discovered the laws of the principal irregularities. Ptolemy, by his observations and researches, determined the principal elements of the lunar motions with much exactness. Horrox, who adopted the discoveries of Kepler, formed, about the year 1640, a theory of the moon, founded partly on his own observations. From this theory, Flamstead, about the year 1670, computed tables which he found gave the place of the moon far more accurately than any other. Flamstead himself soon after furnished observations, by which Sir Isaac Newton was enabled to investigate, by the theory of gravity, the lunar irregularities, which he has given in his ever-memorable work. Notwithstanding the field opened by the publication of the "Principia," and the known necessity of exact tables of the moon for the discovery of the longitude at sea, seventy years elapsed from the publication of that great work before any tables were formed for the moon, which gave its place within one minute. Clairaut made, after Newton, the first considerable advances in the improvement of the lunar theory from the principles of gravitation. Professor Mayer, of the University of Gottingen, first published tables, by which the moon's place might be computed to one minute. About the year 1780, Mr. Mason, under the direction

of Dr. Maskelyne, improved, by considerable alterations and additions, the tables of Mayer. Better tables were afterwards furnished by M. Bürg, of Vienna, which appear to give the place of the moon to less than twenty seconds. The improvements in these tables were founded entirely on the observations of Dr. Maskelyne, for which purpose 3600 places of the moon, observed at Greenwich in the space of about thirty years were used.

The tables of M. Bürg have been superseded by those of Professor Hansen, which are now used in computing *The Nautical Almanac*, and *Conn. des Temps*. They are so accurate as never to exceed an error of ten seconds, the average error being only three.

268. Eclipses of Jupiter's satellites furnish us with ready methods of finding the principal elements of their orbits. Their mean motions about the centre of Jupiter are deduced by observing, after a long interval, the time elapsed between two eclipses of the same satellite, when Jupiter is near opposition. In this manner the mean motion may be attained to with great accuracy. The places of the nodes and the inclinations of their orbits may be found by observing the different durations of the eclipses of the same satellite. Their orbits are all inclined by very small angles to the plane of Jupiter's equator; the greatest inclination is that of the third, which is 5′ 3″. The first two satellites move in orbits very nearly circular, as astronomers have not been able to detect any eccentricity. The third has a variable eccentricity. The orbit of the fourth satellite is more eccentric. The inclinations of their orbits, and the places of their nodes, are variable.

The complete illustration of the motions of the satellites from gravity was long a desideratum in astronomy. The attraction of the satellites to each other principally occasions the difficulty. M. Laplace has, however, fully developed their motions, and furnished Theorems, by which M. Delambre has computed tables which give the times of the eclipses with great exactness.

The three inner satellites of Jupiter return to the same position, with respect to one another, in 437¾

days. Hence this is the period of the irregularities of the three first satellites arising from their mutual disturbance.

269. On the Orbits and periodic returns of Comets.—When a comet appears, the observations to be made for ascertaining its orbit are of its declinations and right ascensions, from which the geocentric latitudes and longitudes are obtained. These observations of right ascension and declination must be made either with an equatorial instrument, or by measuring with a micrometer the differences of the declination and right ascension of the comet and a neighbouring fixed star. The observations ought to be made with the utmost care, as a small error may occasion a considerable one in the orbit. The orbits of the planets being elliptical, it would naturally occur to try whether the motions of the comets are not also in elliptical orbits. But here the difficulty is much greater than for the planets. For the latter we have observations in every part of their nearly circular orbits. For the comets we have observations only in a small part of their orbits, which are very eccentric, and of which many make considerable angles with the ecliptic. Hence to determine the orbit of a comet, from such observations as we can make during its appearance, ranks among the difficult problems in astronomy.

270. Before the time of Newton, astronomers either did not suppose their orbits elliptical, or despaired of being able to determine them from observation. Not long, however, before the publication of the "Principia," M. Doerfell, a German, found that the motion of the famous comet of 1680 might be nearly represented by a parabola, having the sun in its focus. This comet appeared to approach the sun directly, and descended from it again in the same manner.

When the action of gravity was subjected to calculations by the illustrious Newton, the theory of the motions of comets became perfectly understood, and it was concluded that their orbits in general were very eccentric ellipses. But in computing an orbit from

observations, all we are in general able to do is to represent, with accuracy, the comet's motion while in the neighbourhood of the sun, and visible to us. We can do this by supposing the orbit a parabola—that is, an ellipse whose greater axis is infinite, and on that hypothesis computing its elements, in which way we can determine its path with sufficient exactness to make the observed and computed places agree very nearly with each other.[a] It is seldom, indeed, that we can expect to compute the elliptic orbit from the few observations we are enabled to make. We may, it is true, deduce many eccentric ellipses that will represent, with the same accuracy as the parabola, the apparent motion. Were we to attempt to compute the exact ellipse, the necessary errors of observation would render our conclusions quite uncertain. Hence, in general, we have no knowledge of the axis, and consequently of the periodic time, but from former observations of the same comet.

271. There are seven elements which we may consider as determining the identity of a comet: these are the Perihelion distance, the Eccentricity, the place of the Perihelion, the time of Perihelion passage, the place of the node, the inclination of its orbit, and its motion being direct or retrograde. If two comets, recorded in history, are found to agree in these circumstances, there can be hardly any doubt of their identity, and consequently we obtain the knowledge of its periodic time, and are enabled to point out the future appearances of the comet.

272. Halley's Comet.—Dr. Halley found that the comet which he observed in 1682 agreed in these circumstances with that observed by Kepler in 1607, and with that observed by Apian in 1531, whence

[a] Sir Isaac Newton first gave the solution of this problem, which he calls "longe difficillimum." Different solutions have since been given by various authors. The best seems to be that of Laplace. ("Mecanique Celeste," tom. 1, p. 221.)

the *foretold* that it would return again in the latter end of 1758, remarking that it would be retarded by the attraction of Jupiter. Its motion was retrograde, and the elements of the orbit deduced by Dr. Halley, from the observations of Apian in 1531, of Kepler in 1607, and of himself in 1682, were as follows—to these are added the elements deduced from its appearances in 1759 and 1835:—

Passage through Perihelion.				Per dist. Earth's dis. unity.	Place of Perihelion.		Place of Node.		Inclination to ecliptic.	
		D.	H.		°	′	°	′	°	′
1531	Aug.	21	18	.567	301	39	49	30	17	51
1607	Oct.	26	8	.587	302	16	50	21	17	2
1682	Sept.	14	4	.583	302	52	51	16	17	58
1759	Mar.	12	14	.585	303	8	53	45	17	40
1835	Nov.	15	2	.58	304	32	55	10	17	45

This comet was retarded by the action of Jupiter, as Dr. Halley had foretold. This retardation was more exactly computed by Clairaut, who also calculated the retardation by Saturn to be 100 days, in addition to a retardation by Jupiter of 518 days. The result of his computation, published before the return of the comet, fixed April 15 for the time of the passage through Perihelion: it happened on March 12. Dr. Halley's own computation appears also very exact, when it is considered that he did not allow for the retardation by Saturn. This comet returned in 1835. Four different computations fixed that the time of this Perihelion passage would be on either November 4th, 7th, 11th, or 26th of that year. It actually passed that point on November 16. The period of this comet is $76\frac{1}{3}$ years, and its eccentricity ·9674.

A comet was expected in 1789, because one observed in 1532 was supposed to be the same as one observed in 1661. Halley mentioned the probability of their

being the same, but not with confidence, and the event has made it very doubtful whether they were the same.

An ingenious computation has been made by Laplace from the doctrine of chances, to show the probability of two comets being the same, from a near agreement of their elements. It is unnecessary to detail at length the method here. It supposes that the number of different comets does not exceed one million, a limit probably sufficiently extensive. The chance that two of these, differing in their periodic times, agree in each of the seven elements within certain limits, may be computed; by which it was found to be as 1200 : 1, that the comets of 1637 and 1682 were not different, and thus Halley was justly almost confident of its re-appearance in 1759. As it did appear then, and afterwards in 1835, we may expect, with a degree of probability approaching almost without limit to certainty, that it will re-appear in 1911, at the completion of its period. But with respect to the comet predicted for 1789, from the supposition that those of 1661 and 1532 were the same, the case is widely different. From the discrepancy of the elements of these comets, the probability that they were the same is only three to two, and we cease to be surprised that we did not see one in 1789.

The comet that appeared in 1680 is supposed to have been the same with those which were seen in 1105, 575, and B.C. 43. The comet of 1556 has been considered to be the same as that of 1264.

273. LEXELL'S COMET.—A comet appeared in 1770, very remarkable from the result of the computations of Lexell, which indicated a period of only 5½ years; it has not been observed since. There can be no doubt that the periodic time of the orbit which it described in 1770 was justly determined; for M. Burckhardt has since, with great care, re-computed the observations, and his result gives a periodic result of 5½ years.[a] It has been concluded that this comet described

[a] Laplace "Mecanique Celeste." Tom. 4, p. 223.

in 1770 an orbit quite different from its former orbit, and that its orbit was again changed after that year, and this double change is accounted for as follows :—

Lexell had remarked that this comet, moving in the orbit he had investigated, must have been near Jupiter in 1767, and would also be very near it again in 1779, from whence he concluded that the disturbing attraction of Jupiter at its former approach changed the Perihelion distance of the orbit, by which the comet became visible to us; and that, in consequence of the latter approach, the Perihelion distance was again, by the same attraction, increased, and so the comet again became invisible, even when near its Perihelion. In 1767 it was so near Jupiter that Jupiter's attraction on the comet must have been three times that of the sun, and this disturbing action lasted for several months. In 1779 the distance of the comet from Jupiter was only $\frac{2}{491}$ of its distance from the sun, when the planet's attraction was 230 times that of the sun. This explanation has been in a manner confirmed by the calculations of Burckhardt, from formulas of Laplace, and afterwards by Leverrier. Burckhardt has found that before the approach of Jupiter, in 1767, the Perihelion distance might have been 5·08, and that, after the approach in 1779, it may have become 3·33. the earth's distance being unity. With both these Perihelion distances the comet must have been invisible during its whole revolution. The Perihelion distance of the temporary orbit in 1770 was 0·67. This comet is called the *Lost Comet*, because it was never seen before or since that year.

274. This comet was also remarkable by having approached nearer the earth than any other comet that has been observed; and by that approach having enabled us to ascertain a limit of its mass or quantity of matter. Its distance, in July, 1770, was less than a million and a half of miles. Laplace has computed that if its mass had been equal to that of the earth, it would have shortened the length of our year by $\frac{1}{9}$ of a day. Now it has been perfectly ascertained, by the computations of Delambre on the Greenwich observa-

tions of the sun, that the length of the year has not been changed in consequence of the approach of that comet by any perceptible quantity, and thence Laplace has concluded that the mass of that comet is less than $\frac{1}{5000}$ of the mass of the earth.

275. Encke's Comet.—The comet which takes the shortest time to revolve round the sun, and which is most frequently seen by us, is that of Encke. It is a telescopic comet, but is sometimes seen by the naked eye. Comets having appeared in 1786, 1795, 1805, and 1818, which were then supposed not to be the same, circumstances led Professor Encke of Berlin, in 1819, to suspect that these were different appearances of one comet, of a short period; and after elaborate calculations, he showed that this was the case; and that the period of this comet was 1207 days, or about $3\frac{1}{3}$ years. He predicted that it would return in 1822; and the re-appearance of the comet in that year, and in recurring years since, showed that his calculations have been correct. Its eccentricity is .8474; Longitude of Perihelion, 158° 1′; Longitude of Ascending Node, 334° 31′; Inclination of Orbit, 13° 5′ Perihelion distance, .3382 (less than that of Mercury). Its greatest distance from the sun is $\frac{4}{5}$ of that of Jupiter.

The most interesting question connected with this comet arises from the fact, that its periodic time is found to diminish slowly, by about $\frac{1}{8}$ of a day in each revolution ; this would indicate (from Kepler's law) that its mean distance from the sun was gradually diminishing, and that the comet would ultimately fall into the sun. No reason, connected with the law of gravitation, could account for this fact. It was consequently suspected by Encke, that the planets and comets moved round the sun, not in free space, but in some kind of resisting medium of extreme tenuity, acting sensibly upon the motion of a body of such small density as the comet. Such resistance would diminish the velocity, and therefore the centrifugal force, and the sun's attraction would then diminish its distance. We have not the means of knowing whether other comets are similarly affected, inasmuch as there are but two others,

Halley's and Faye's, whose returns have been carefully observed.

Encke's comet approaches to the sun within the orbit of Mercury; consequently, the disturbing action of the attraction of this planet upon the comet has enabled us to calculate the mass of Mercury which is proportional to that force.

276. FAYE'S COMET was discovered by M. Faye, of Paris, in 1843, who computed that it would return in less than $7\frac{1}{2}$ years. Le Verrier predicted its return on April 3, 1851, and it reached its Perihelion within a day of that time, it has since appeared in 1858 and 1865; and the comparison of its observed with its calculated position, do not lead us to think that the existence of any resisting medium can be traced in its motions. The observations with regard to Halley's Comet have not enabled astronomers as yet to pronounce as to its being affected by a resisting medium.

277. BIELA'S COMET.—Another comet of short period, and whose orbit is comparable in many respects to those above named, was identified by M. Biela, a Bohemian, in 1826, with a comet which had appeared in 1772 and 1805. He calculated that it had a period of $6\frac{2}{3}$ years, and it has since been recognised, on its reappearance, as predicted, in 1832, 1846, and 1853. The Perihelion distance of this planet is 0.854 (the earth's distance being unity), or about $\frac{7}{8}$ of the earth's distance from the sun.

This being the case, since the longitude of the Perihelion of Biela's comet is 109° 6′, if the comet happened to pass its Perihelion on the 30th November, the day on which the earth is in that part of its orbit, it would enclose the earth in its nebulosity. When this comet appeared in 1832, great fears were entertained that this would be the case; the comet, however, passed the Perihelion a month earlier, and consequently got out of the way of the earth before its arrival in the same longitude. One of the most remarkable circumstances connected with Biela's comet, was first observed in 1846, when it appeared as a double comet, one being of permanent brightness, while the other was first very faint, then increased to an equality in brightness with

the first, and then became gradually more faint, until it disappeared some weeks before the other. A similar appearance was presented on its return in 1852. By tracing the orbits of these comets backwards, it has been found that, in September, 1844, they were at a distance from each other less than twice the diameter of the earth, from which it has been supposed by some that they were originally one comet, which, from some internal cause, became divided into two. Biela's comet is so small as to be seen with difficulty by the naked eye.

278. In addition to Encke's, Faye's, and Biela's comets, there are three others of short period, whose return has been predicted and verified by observation.

D'ARREST'S COMET (telescopic) has a period of 6 years and 140 days, its distance from the sun at Perihelion is 111 millions of miles, and it has an eccentricity of 0.661. Brorsen's comet (telescopic) has a period of $5\frac{2}{3}$ years, a Perihelion distance of 62 millions of miles, and an eccentricity of 0.8023, and Winnecke's (or Pons') comet, has a period of $5\frac{1}{2}$ years, and a Perihelion distance of 73 millions of miles, with an eccentricity of 0.755. These six comets describe ellipses, whose eccentricity varies from a little more than $\frac{1}{2}$ to a little more than $\frac{4}{5}$, and their orbits lie inside that of Saturn, and do not much extend beyond the orbit of Jupiter. Faye's comet, which has the greatest Aphelion distance of the six, extends beyond the Aphelion distance of Jupiter only 45 millions of miles.

279. Very great interest was excited in the year 1843 by the appearance of a comet, consisting of a tail of nearly 60° in length, attached to a head and nucleus of such great splendour as to have been actually visible in full sunshine. The perihelion distance of this comet being 0·00534, is the smallest on record, and exceeds the radius of the sun by only one-seventh of its whole distance. Some have supposed that this comet has a period of 175 years, being identical with the great comet of 1668.

CHAPTER XVI.

APPLICATION OF ASTRONOMY TO GEOGRAPHY—MEASUREMENTS OF DEGREES OF LATITUDE.

280. ASTRONOMY furnishes several methods of finding latitudes and longitudes on land. But the latter are found with much greater trouble and less accuracy than the former. The methods of finding the latitude of a place by observations made by the larger instruments have been before mentioned, and it will here be only necessary to take notice of the use of Hadley's sextant for this purpose. By means of this portable instrument, the latitude may be found from observations of the sun's meridian altitude, with a degree of accuracy sufficient for many purposes of geography.

281. At sea, the horizon is generally sufficiently defined to serve for measuring the sun's altitude by Hadley's sextant; but on land, an artificial horizon is necessary, that is, we must make use of an horizontal reflecting surface, by which an image of the sun may be formed by reflection. We measure, by the sextant, the angular distance between the upper or lower limb of the sun and its reflected image, which distance is twice the altitude of the limb, because the rays of light are so reflected that the angles of incidence and reflection are equal.

There are various methods of forming this artificial horizon. Mercury and water afford the most convenient horizontal surfaces, when sheltered from the agitation of the air. For general use, perhaps, water ought to have the preference.

282. With respect to the longitudes of places on land, our means of obtaining accuracy are much greater than

at sea. We can repeat our observations at our leisure, and use such observations only as admit of the greatest precision. From the present state of Geography, as to the more known parts of the world, it cannot be much advanced by the lunar method of obtaining the longitude.

An occultation of a fixed star by the dark edge of the moon, observed at two places, the longitude of one of which is known, affords the greatest precision; because this phenomenon is instantaneous.

Eclipses of the sun rank next, but are not quite so accurate, because the beginning and end of an eclipse of the sun cannot be observed so exactly as the occultation of a star by the dark edge of the moon. The transits of the inferior planets also afford much accuracy.

The observations, however, which occur most frequently are the eclipses of the satellites of Jupiter. The first satellite, passing more quickly into the shadow of Jupiter than the others, is best adapted for this purpose. By taking a mean of the results of the observations made on the first satellite, both in its immersions and emersions, great accuracy can be obtained.

283. By the assistance of a transit instrument, the longitude of a place can be had from observation of the difference of the times of passages of the moon and a fixed star, compared with the difference observed at Greenwich, or in some place of known longitude.

Method of finding the difference of longitudes of two places on land by Moon-culminating stars.—The moon moves among the fixed stars, as we have seen, at the rapid rate of one minute of space in two minutes of time; consequently, her change of right ascension is considerable. If we select a star which has nearly the same declination as the moon, and does not differ much in right ascension, and note the interval in time between the transits of the bright limb of the moon and that star across the meridian of a place, and if we know the interval in the corresponding times of transit as seen from Greenwich, we know the increase of the moon's

right ascension between her crossing of the meridian of Greenwich and that of the place; and, knowing the rate per hour at which the right ascension changes, we can find the time which elapsed between the transits of the star at the two places, and consequently the difference of longitudes. We may know the interval of time between the transits of the moon and the star as seen at Greenwich, either by actual observation, or by computations founded on the moon's motion as given in the Nautical Almanac. The star is selected so as to have the same declination as the moon, in order that both may be equally affected by instrumental errors.

The computations for occultations, for transits of the inferior planets, and for eclipses of the sun, are long and complex. This arises from the effects of parallax, the phenomena not being observed at the *same* instant by each observer.

284. The only difficulty, whether at sea or land, for finding the longitude, is to ascertain the time at some place where the longitude is known. This may be ascertained for near places, as well by terrestrial signals, as by celestial observations. An eclipse of a satellite of Jupiter may be compared to a signal. An explosion or an instantaneous exhibition or extinguishment of a light being observed at two places, and the time noted exactly at each when it took place, the difference of longitudes will be had by simply taking the differenee of the times. In this manner considerable assistance has been afforded to Geography. The difference of longitude between two places on the earth's surface can now be found by means of the electric telegraph. If we wish to find the difference of time (D) between two stations whose distance in miles is l, then when a star crosses the meridian at the eastern station an electric signal is sent to the western station; if T be the time at the eastern station and t at the western, and v the velocity of electricity, $T + \frac{l}{v} - D = t$ (1): similarly when a star crosses the meridian at the western station, the time t' is tele-

graphed to the eastern, and the observer there notes the time of arrival T′; then $t' + \mathrm{D} + \frac{l}{v} = \mathrm{T}'$ (2), subtracting (1) from (2) we get 2D; adding them we get $\frac{2l}{v}$, and hence the velocity of the electric current.

285. But the mere knowledge of the latitudes and longitudes of places is not sufficient for the Geographer. The exact figure and exact magnitude of the earth are also necessary in order to ascertain the exact distances of places, to describe and to plan the several countries.

On the hypothesis of the earth being a sphere, nothing more is necessary toward ascertaining its dimensions than *to measure the length of a degree of latitude:* that is, to determine the length of an arc of a terrestrial meridian, the latitudes of the extremities of which differ by one degree. The mode of ascertaining this is easily understood.

The difference of latitude of two places in nearly the same meridian is to be ascertained by celestial observations. The distance on the meridian, between these two places, is to be obtained by terrestrial measurement. A horizontal base line, of a few miles in length, is to be measured in a convenient situation, and this base is then to be connected with the two places by forming a series of triangles, the angles of which are to be measured by a proper instrument, and then the distance of the two places computed by trigonometry.

286. Let Q and T (Fig. 44) represent two places nearly in the same meridian QM: the line AC the base, the length of which is ascertained by actual measurement. The angles of the triangles ACH, APH, NPH, PNQ, also of CHK and CTK are to be ascertained by an instrument adapted for taking angular distances. Two angles of each triangle would be sufficient, as from thence the third angle is known: but to verify the observations it is usual to observe all the angles of each triangle.

The base AC and the angles of the triangle ACH being known, the other sides AH and HC are had by computation, and thence the sides of the triangles APH, PHN, PNQ, CHK, and CTK.

From T draw TMG perpendicular to the meridian QM, also let DQ, PE, and CF be perpendicular to QM, and PD, AE, AF, and CG parallel to the same.

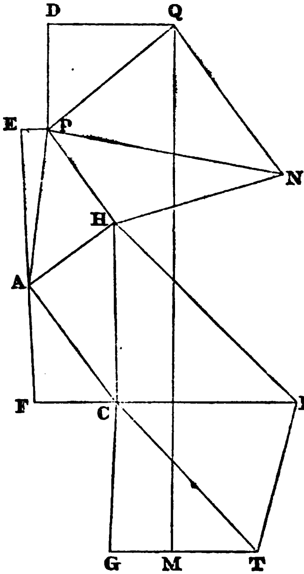

Fig. 44.

Now QM = DP + AE × AF + CG. The sides PQ, PA, &c., being known, PD, AE, &c., will be had by the solution of right angled triangles, provided the angles DQP, EPA, &c., are known. These angles will be known if the angle PQM, or the angle that the direction of one of the stations P seen from Q makes with the meridian, be known. This angle may be obtained by different methods.

The sun being observed in the same vertical circle as the object P, the azimuth of the sun may be computed from the latitude of the place, the declination and distance in time of the sun from the meridian: thus the azimuth of P, or the angle PQM, will be had.

The pole star, when near its greatest elongation from the meridian, changes its azimuth very slowly, and therefore is very convenient for ascertaining the direction of the object in respect to the meridian. The differences between the azimuth of the pole star, when at its greatest elongations east and west, and the azimuth of the object being obtained, half the sum or difference of these will be the azimuth of the object.

It is evident that when the inclination of PQ to the meridian is known, the inclinations of PA, AC, &c., to the meridian and its parallels will also be known, because the inclinations of these lines to each other are known.

The observations being made for ascertaining the length of QM, the difference of latitudes of the stations Q and P is to be observed with the utmost accuracy, by means of a zenith sector or other instrument affording sufficient exactness.

For this purpose the zenith distance of a star near the zenith is to be observed at each place, and the sum or difference, according as the star is on a different or on the same side of the zenith at each place, will give the difference of latitude. The changes in the apparent place of the star between the observations, arising from aberration, &c., must be taken into account.

The length of the arc of the meridian, corresponding to a known difference of latitude, being thus found, the length of one degree will be had by a simple proporportion.

287. The minute particulars that must be attended to, in order to obtain the greatest accuracy, cannot be enumerated here. They are to be met with in the several accounts of the modern measurements.

If the instrument used in measuring the angles give the angular distance, and not the horizontal angular distance between the objects, the elevations or depressions must be also observed, that the horizontal angles may be computed.

The triangles formed are not plane triangles, but spherical triangles not differing much from plane. The sum of the three angles of each is therefore somewhat more than 180°; but this excess is easily computed, and therefore the sum of the three angles may be still used for verification.

The computations of spherical triangles being more difficult than of plane triangles, mathematicians have devised ingenious methods to reduce the computation of

these spherical to plane triangles, being assisted by the small difference between them and plane triangles.

288. The results of different measurements have shown that the degrees towards the poles are longer that those nearer the equator; and therefore that the earth is not exactly a sphere. This will be better understood by a short account of the principal steps by which we have arrived at our present knowledge of the form and dimensions of the earth.

289. The first modern measurement distinguished by a tolerable degree of accuracy is that of Norwood, in 1635. He ascertained the difference of the latitudes of London and York, and then measured their distance, allowing for the turnings of the roads and for the ascents and descents. From which he deduced the length of a degree = 122,399 English yards. According to the latest determinations it should have been = 121,660 yards.

At this time no circumstances were known which could tend to a knowledge of the exact figure of the earth.

In the year 1671 it was discovered, by a comparison of the times of vibrations of the pendulums at Cayenne and Paris, that the weights of bodies were less near the equator than at Paris. From whence Huygens considered it probable that the form of the earth was not spherical, but that it was a figure formed by the revolution of an ellipse about the lesser axis. Sir Isaac Newton, arguing from juster principles than those of Huygens, was also led to the same conclusion, and actually computed the ratio of the equatorial and polar diameters on the hypothesis of the earth having been at first a homogeneous fluid, revolving on its axis. The ratio of the equatorial to the polar diameter he found to be 230 : 229. At this time, 1686, no evidence from actual measurement existed, but Newton lived till it was ascertained by observation that the ratio of the polar and equatorial diameters of Jupiter was *nearly* such as his theory gave on the hypothesis of an uniform density. He also lived till the results of actual

measurements made in France appeared entirely inconsistent with the form which he had assigned. Subsequent measurements, made soon after Newton's death, fully established that the equatorial exceeded the polar diameter.

290. Picard, in 1670, measured an arc of the meridian, commencing near Paris and extending northward, and found, in latitude $49\frac{1}{2}°$, a degree = 121,627 yards, differing only by about 35 yards from what is now considered as the most exact length. This accuracy seems to have been accidental, and obtained by a compensation of errors.

A few years afterwards, by order of the French King, Cassini, assisted by several other astronomers, undertook the measure of the whole arc of the meridian, extending through France from Dunkirk to Collioure. This work was finished in 1718. Among the results obtained, it was found that in latitude 46° a degree of the meridian =121,708 yards, and in latitude 50° =121,413.

Thus the degrees appeared to diminish as the latitude increased, instead of the contrary. For it is evident that if the curvature of the earth diminish as we recede from the equator towards the poles, the degrees of latitude ought to increase, because the less the curvature, the greater space must be gone over to change the elevation of the pole by one degree. This result, therefore, appeared to contradict Newton's conclusion, that the earth was nearly an oblate spheroid, that is a solid, formed by the revolution of an ellipse about its lesser axis. To support Newton's conclusion, it was objected that these degrees were so near each other that the errors of observation and measurement might greatly exceed the difference of degrees that would come out from computation by Newton's figure. But this mode of getting over the difficulty was not satisfactory. It was still contended by some of the French Academicians that the polar diameter of the earth was greater than the equatorial.

To remove all doubt, it was proposed that two de-

grees should be measured, one as near to the equator, and the other as far northward, as conveniently could be done.

Accordingly, in 1736, a company of French and Spanish astronomers went to Peru, to measure an arc near the equator; and a company of French and Swedish astronomers undertook to go to Lapland, and measure an arc near the Arctic circle.

The interesting particulars of their labours and difficulties have been minutely described by themselves, and their exertions for attaining the utmost accuracy cannot be sufficiently admired.

From a comparison of the measurements in Peru and in France, the equatorial diameter[a] appeared to exceed the polar by about $\frac{1}{304}$ part of the whole.

From a comparison of the measurements in Lapland and in France, the excess appeared to be $\frac{1}{210}$.

Thus the principal point was settled, that the earth was flatter towards the poles; but the quantity of that flatness seemed by no means ascertained. The measures in Lapland and in Peru seemed quite discordant. But from several circumstances, greater confidence was placed in the measure in Peru than in Lapland; although the latter seemed executed with all due care.

291. Arcs of the meridian have since been measured in several countries: but till this century no satisfactory conclusion was drawn respecting the degree of ellipticity in the earth, and even now greater exactness is desired.

In the year 1787, it was determined to connect the observatories of Greenwich and Paris by a series of

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[a] If the density of the earth were uniform, and if the earth had been originally in a fluid state, its form would be accurately that of a spheroid, generated by the revolution of an ellipse about its minor axis. The proportion of its diameters would then be readily investigated from a comparison of the lengths of two degrees of latitude (Vince's Astronomy, Vol. ii. p. 98). As, however, the exact form of the earth is not known, the investigation of the proportion of the diameters from the comparison of two degrees of latitude is only to be considered as a near approximation.

triangles: and to compare the differences of longitudes and latitudes, ascertained by astronomical observations, with those ascertained by actual measurement. The late Major-General Roy conducted the British measurement. The British Triangles were connected with those of the French, by observations made across the Straits of Dover. In this manner, assuming the latitudes of the respective observations, as had been previously ascertained, it was found that in latitude 50° 10′ a degree of the meridian was 121,686 yards.

The measurement in England, which was begun with a reference only to the relative situations of the observatories of Greenwich and Paris, was extended to a survey of the whole kingdom. This, General Roy having died, was conducted by Colonel Mudge, with great skill and assiduity. In the course of his survey, in the year 1801, he measured an arc of the meridian between Dunnose in the Isle of Wight, and Clifton in Yorkshire. The difference of latitude (nearly three degrees) was ascertained by an excellent zenith sector made for the occasion.

From this measurement it resulted that the length of a degree in latitude 52° 2′ = 121,640 yards.

292. An arc of the meridian of nearly 10° in length has been measured in India, between a station near Cape Comorin, in lat. 8° 9′ and a station in the Nizam's dominions in latitude 18° 3′. This has been achieved by the exertions of Major Lambton, continued during several years. He was furnished with excellent instruments, similar to those used by Colonel Mudge. The result of Major Lambton's measurement gives 120,975 yards for the length of the degree in latitude 13°. 3′ N.

A comparison of the degrees ascertained by Colonel Mudge and Major Lambton gives the excess of the equatorial above the polar diameter = $\frac{1}{310}$.

293. At the time the English measurement was going on, the French astronomers, Mechain and Delambre, engaged in measuring the arc of the meridian from Dunkirk to Barcelona, which places are nearly under

the same meridian, and differ in latitude by about $9\frac{1}{2}°$. Their operations commenced in 1792, and after struggling with the greatest difficulties, arising from the unhappy situation of their country, they succeeded in accomplishing the object of their labours. From this measurement, compared with the measurement near the equator in 1736, &c., they deduced the excess of the equatorial above the polar diameter = $\frac{1}{334}$.

294. In the year 1802, M. Swanberg and other Swedish astronomers undertook to repeat the operations of the French Academicians, which they had performed near Tornea in Lapland, in 1736. This was an object of considerable importance, on account of the different results deduced from the comparisons made with the measurements in France and Peru.

M. Swanberg has given a most able detail of this operation and of the computations. The result which he deduces from a comparison with the new measurement in France, is an excess of the equatorial above the polar diameter = $\frac{1}{307}$.

A comparison of the measurement of Major Lambton and of his own gives the excess the same, viz., $\frac{1}{307}$.

Other comparisons incline him to fix the most probable excess at $\frac{1}{323}$.

The discordance of the degree measured in Lapland in 1736 and 1802 led to an examination of the source of the difference; and it appeared that the French Academicians had erred ten or eleven seconds in the latitude of one of their stations. All their other measurements were verified. This error was sufficient to account for the difference of results.

295. The operation of measuring a degree of latitude consists in ascertaining the length of the arc of the meridian, and in ascertaining the difference of latitudes of the extremities. The latter part is not susceptible of near so great accuracy as the former. A second in latitude answers to about 33 yards, and the difference of latitude cannot be probably ascertained nearer than two seconds, supposing no cause of irregularity to affect

the plumb line. But there is sufficient proof that the plumb line is sometimes displaced several seconds by the attraction of mountains or ofdifferent strata. Colonel Mudge and the French astronomers experienced this, in a considerable degree.

The terrestrial measurements are susceptible of great accuracy. It is usual to measure a base of verification as far distant from the first base as can conveniently be done, and then compare this base with its length deduced by computation, from the first base and the angles measured: this was done by the French astronomers in their late survey. The length of the base of verification measured was upward of 7 miles, and at the distance of above 400 miles from the former base, and yet it did not differ by 12 inches from the length inferred by computations. In the Ordnance Survey the computed lengths of the base did not differ from the measured lengths by three inches in five cases out of six.

296. The instruments used in the English measurement, and in that by Major Lambton, were a steel chain, an instrument for taking horizontal angles, the circles of which were 3 feet in diameter, and a zenith sector. Mr. Ramsden exerted his great talents in making the construction of these instruments as perfect as possible.

The first base in the English measurement was above five miles in length, and was measured in 1787 by glass rods: it was again measured in 1791 by the steel chain, and the two measurements differed only by about 3 inches.

The instrument for taking the angles, sometimes called Ramsden's Theodolite, besides the accuracy it afforded, gave at once the horizontal angles, in which it had a great superiority over the instruments by which the angular distances between the stations were taken, and which afterwards required to be reduced by computation to the horizontal angles.

297. In the measurements in France and Lapland, a repeating circle, of which the radius was only a few inches, was used for taking the angles and making the

observations for the difference of latitudes of the extremities of the arcs. However inadequate at first sight such an instrument may appear to obtain conclusions in which extreme accuracy is required, it must be allowed that it fully answered the purpose for which it was intended. The length of the computation was much increased, as the angles observed were to be reduced to the horizon, and other reductions made: but these inconveniences seem much more than compensated by the portableness of the instrument.

The French base was measured by rods of platina: the Swedish by rods of iron: the requisite allowance was made for the changes of temperature during the operations.

The Irish base for the calculations of the Trigonometrical Survey was measured at Lough Foyle by means of a combination of two parallel bars, the lower of brass and the upper of iron, connected at the middle; as the expansion of brass from heat is to that of iron as 5 to 3, it is easy to see that if perpendiculars at the extremities of this compound bar be produced, so that the whole line produced be to the produced part in this ratio, their extremities will remain unaltered in position by the expansion of the bars.

298. The result of the measurement in France has been used to ascertain a standard of measure. The length of a quadrant of the meridian was computed, and found to be 5,130,740 toises, or 10,936,578 English yards. This was divided into ten million parts, and one part, which was called a *metre*, was made the unit of measure. All other French measures are deduced decimally from this. The French metre then is 10,936,578 yards, or 39·37 inches nearly.

Computing from the length of the degree in latitude 45° the mean diameter of the earth comes out 7912 English miles nearly, and the equatorial diameter will exceed the polar by about 26·471 miles,[a] the equatorial

[a] A relation of the measurement in Lapland in 1736 was published by Maupertuis, and also by the Abbé Outhier, which is more minute

diameter being 7,925·604 miles, and the polar diameter 7,899·114 miles; this is the result given by Bessel. Airy's is very nearly the same; he makes the former to be 7,925·648 miles, and the latter 7,899,170 miles, and the proportions nearly as 300 to 299. In our latitude there are very nearly 365,000 feet in a degree of the meridian.

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than that of Maupertuis (vide Conn. des Temp. 1808). Separate accounts of the measurements in Peru were published by Ulloa, Bouguer, and Condamine.

A very particular account of the measurements in France was published by Cassini, in 1744.

The particulars of the recent measurement in France have been published by Delambre, and of that in Lapland by Swanberg (vide Conn. des Temps. 1808).

An account of the measurement by General Roy will be found in the Phil. Trans. for 1787 and 1790. Of that by Col. Mudge in the Phil. Trans. for 1803.

The latest account of Major Lambton's measurement is given in the Phil. Trans. 1818, p. 2.

An interesting account of the different measurements is also given under the article "Degree," in Rees's Cyclopædia.

CHAPTER XVII.

ON THE CALENDAR.

299. Among the different divisions of time, the *civil year* is one of the most important. The *solar year*, or the interval elapsed between two successive returns of the sun to the same equinox, includes all the varieties of seasons.

The civil year must necessarily consist of an exact number of days; but the solar year consists of a certain number of days, and of a part of a day (Art. 246). Hence an artifice is necessary to keep the commencement of the different seasons as nearly as possible in the same place of the civil year: that is, if the sun enter the equator on the 20th of March in one year, that it may always enter it on the same day, or nearly on the same day, and that the solstices may be always as nearly as possible on the same day.

The common civil year consists of 365 days; the solar year of 365 days, 5 hours, 48 minutes, and 50 seconds, or 365 days, 6 hours nearly.

It is evident that if each civil year were to consist of only 365 days, the seasons would be later and later every year, and in process of time change through every part of the year.

300. In the infancy of astronomy it was not to be expected that the exact length of the solar year could be obtained with much accuracy, and we find the Egyptians, and other nations, availing themselves of another method by which they regulated the time of their agricultural labours. They observed when Sirius or Arcturus, or some other bright star, after it had been obscured by the splendour of the solar rays, first

became visible in the east before sunrise. This is called the heliacal rising of a star. From this time they reckoned a certain number of days to the commencement of the respective seasons of ploughing and sowing, and of other labours in husbandry.

In this manner they dispensed with an exact knowledge of the length of a year. They were ignorant of the precession of the equinoxes, which in a few centuries would have occasioned their rules to fail, or rather to change.

301. The first useful and tolerably exact regulation of the civil year, by help of the solar, took place in the time of Julius Cæsar. It was then provided that every fourth civil year should consist of 366 days, and the addition of the day should be made "die sexto calendas martias," whence the term *bissextile* applied to the year that consists of 366 days. We usually call it leap year, and the additional day is called the 29th of February.

The Calendar so ordered was called the Julian Calendar.

302. By the Council of Nice, held in the year 325, it was fixed that the feast of Easter, by which the movable fasts and festivals of the Church are regulated, should be the first Sunday after the first full moon, which happened on or after the 21st of March. At that time the equinox happened on the 21st of March. Thus the festival of Easter was intended to be regulated by the spring equinox.

303. At that time it must have been known that the excess of the solar year above 365 days was not quite six hours, and that, therefore, in using the Julian Calendar, the equinox would happen sooner every year. There, however, seems to have been no provision made on that account.

The true length of the solar year being less than 365 days, 6 hours, by 11 minutes nearly, the equinox every fourth year was nearly 44 minutes earlier, and in course of time the 21st of March, instead of being the day of the equinox, might have been the day of

the summer solstice. Thus the fast of Lent and festival of Easter might have been observed in the middle of summer.

This inconvenience was foreseen before any material alteration had taken place. In the time of Pope Gregory, in 1577, the equinox happened on the 11th of March, or ten days before the the 21st. It was then determined to remedy the error that had already taken place, and to provide against a future accumulation.

It must be generally allowed that it was right to guard against an increase of the error, but it may be doubted whether a greater inconvenience did not take place to the people in general by correcting the error of the ten days, than if it had remained.

304. The 5th of October, 1582, was called the 15th, and thus the equinox was restored to the 21st of March.

A recurrence of error was prevented in the following manner. The true length of the solar year, as far as it was then known from the best tables, founded on the observations of Copernicus, Ptolemy, and Hipparchus, was 365 days, 5 hours, 49 minutes, and 16 seconds. By adding a day every fourth year, in four years the addition was $4 \times (10^m\ 44^s)$ too much, or the accumulation of error in 400 years $= 400 \times (10^m\ 44^s)$ $= 2$ days, 23 hours, and 33 minutes nearly. Hence if, instead of making *every* fourth year leap year, every hundredth year for three centuries successively be made a common year, and the fourth hundred year be a leap year, the error in 400 years will be only about 27 minutes, and therefore the error in 20,000 years would not be more than a day.

Hence the correction adopted by Pope Gregory, that the years 1700, 1800, 1900, 2100, 2200, 2300, 2500, which, by the Julian Calendar, are leap years, should be common years, and that the years 2000, 2400, &c., should remain leap years, is quite sufficient. The more correct length of the solar year, as now determined, proves the Gregorian correction less exact, but not materially so.

305. The Gregorian, or the new style, was not adopted in Protestant countries till a considerable time had elapsed. When it was adopted in England, in the year 1752, the error amounted to eleven days. This was remedied by calling the 2nd of September, 1752, the 13th.

The effect of thus putting, as it were, the seasons backward by eleven days, must at the time have been disagreeable. That our mode of reckoning time was made the same as that of other nations was doubtless a convenience. But it might have been more comfortable to our climate, and the original notions of the festival of Easter, which regulates the other movable fasts and festivals of the Church, if the error that had already accumulated from the Julian Calendar had remained, and the Gregorian correction against future error had been only adopted.

The early climate of Italy might have principally induced Pope Gregory to bring back Easter to the regulations of the equinox: and it may have been a powerful motive in Russia for not adopting the Gregorian alteration in the style, that by retaining and suffering the errors of the Julian Calendar to accumulate further, the fast of Lent and festival of Easter will fall at times more convenient in respect to their seasons.

The year 1800 having been by the Julian Calendar a leap year, and by the Gregorian a common year, the Russian date is now 12 days behind that of the other countries of Europe.

306. The time of the festival of Easter depends on the first full moon on or after the 21st of March, and therefore, strictly, recourse should be had to astronomical calculation to ascertain the time of Easter for each year. But it is sufficient for this purpose to use the Metonic Cycle (Art. 130), the numbers of which are called Golden Numbers.

Short rules and brief tables are given in the Act of Parliament for changing the style, and are usually prefixed to the Book of Common Prayer, by which

the times of Easter may be found for any number of years to come. The computation so made must sometimes differ from what a more exact calculation would give, and the time of Easter, if exactly computed, may vary considerably from the computations founded on the Metonic Cycle. However, as the latter mode of calculation is prescribed by the Act of Parliament (24 Geo. II. c. 23), no inconvenience, from uncertainty as to the time in which the festival of Easter is to be observed, can arise.

By exact computation the 1st of April, 1798, should have been Easter Sunday, whereas by the Calendar prescribed it was not celebrated till the Sunday after. Also the 29th March, 1818, should have been Easter Sunday, instead of the 22nd of March, as found by the prescribed mode of calculation.

The exact lengths of the three several kinds of years are as follows:—

Sidereal year (being the time the sun takes to complete the Ecliptic and return to the same exact position among the fixed stars), $365^d\ 6^h\ 9^m\ 9{\cdot}6^s$.

Tropical or *Solar* year, $365^d\ 5^h\ 48^m.49{\cdot}7^s$.

Anomalistic year (being the interval between two successive arrivals of the earth at perihelion, which *advances* $11''{\cdot}8$ each year among the fixed stars), 365^d $6^h\ 13^m\ 49{\cdot}3^s$.

CHAPTER XVIII.

MASSES OF THE EARTH, SUN, AND PLANETS—THE TIDES—DENSITY OF THE EARTH.

307. In order to find the mean density of the earth, and consequently its mass, we must compare it, as to its attractive force, with a mass of matter of known volume and density. This has been effected in different ways, and they have produced results which do not much differ from each other.

1°. The first method is that of Maskelyne, who ascertained the amount which the attraction of the Scotch mountain called Schehallien caused the plumb-line to deviate from the vertical. Two stations, at 4000 feet distance on the northern and southern sides of the mountain, were selected. Since the direction of gravity is towards the centre of the earth, if the mountain exercised no attraction on the plumb-line, the vertical lines at the two stations should make an angle with each other proportional to the distance between the places, or in this case 41″. A plumb-line attached to a zenith sector was fixed at each station, and the distance from the apparent zenith to a star was measured at both places when the star crossed the meridian. The difference of these zenith distances (or the angle between the plumb-lines) was found to be 53″, or 12″ more than it ought to be; this could only be accounted for by the attraction of the mountain on the plumb-lines, the sum of which at the opposite sides must consequently be 12″. An exact survey had been made of the mountain, and its attraction on the plumb-lines computed from its volume, supposing that its density was the same as the mean density of the earth;

when it appeared that such a density would produce a corresponding disturbance of the plumb-lines of 21″. Consequently the density of the mountain was less than the mean density of the earth, as 12 to 21. From examining the rocks of which the mountain consists, its mean density was found to be 2·8 times that of water. This would give a mean density of the earth equal to 4·9 that of water. From similar experiments made in 1855, at Arthur's Seat, Edinburgh, the mean density of the earth is found to be 5·32.

2°. The second method of determining the mean density of the earth is that which was first adopted by Cavendish, and afterwards by Reich and Baily. Cavendish compared the attraction of the earth with the attraction of two leaden balls of about twelve inches in diameter. Two small balls, each about two inches in diameter, were attached to the ends of a wooden rod, six feet long, which was suspended in a horizontal position by a very fine wire. When the whole apparatus was in a position of equilibrium, the large balls of lead were brought near the smaller balls on opposite sides of the horizontal rod, as applied to each, so as to make the attraction (if any) of the lead on each ball to act in turning the rod in the same direction. A telescope was directed to it from a distance, and its motion measured; the position of the balls was then reversed, and an opposite effect produced. By a comparison of the effect produced by the attraction of the leaden balls on the smaller balls, with the effect of the earth's attraction as exercised upon one of the smaller balls, suspended as a pendulum, we are able to estimate the ratio of the masses of a large ball of lead and of the earth, and, knowing their volumes, we can compare their densities; the specific gravity of lead is known, and hence by an average of results we obtain the mean density of the earth to be $5\frac{1}{2}$ times that of water.

Other experiments have been instituted to effect the same object. From a comparison of the vibrations of the same pendulum at the top and bottom of a mine 1256 feet deep, the mean density of the earth is found by Mr. Airy to be 6·56 that of water. In this case it

must be remembered that the exterior spherical shell of the earth, 1256 feet thick, exercises no attraction on the pendulum at the bottom of the mine.

By a comparison of the length of a seconds pendulum on the summit of a mountain, with that at the sea level, it is found to be 4·84. Newton had estimated it in 1680 to be between 5 and 6.

We may estimate the mean specific gravity of the earth to be 5·46, and the number of cubic miles in its volume to be 259,800,000,000; while the weight of a cubic foot of water is 354 lbs. 6 oz.; hence the earth's weight may be found in the ordinary manner.

308. We have now to show that the attractive force which pulls the moon towards the earth from the tangent to her orbit, is exactly the same as that which makes a stone to fall to the ground. By calculating from the velocity of the moon, and the radius of its orbit, we find the versed sine of the arc described in one second, or the deflection from the tangent in consequence of the earth's attraction, to be 0·0536 of an inch. It is found that at the earth's equator, 3959 miles from the earth's centre, a stone falls $192\frac{1}{2}$ inches in a second. Since the force of gravity is known to diminish in the inverse ratio of the squares of the distances from the centre, the same stone, if removed to the moon's distance, ought to fall 0·0537 in a second, as shown by the following proportion :— $59{\cdot}964^2 : 1^2 : : 1927 : 0{\cdot}053$. The difference of these results is accounted for by the moon's action on the earth, and the sun's disturbing force on the moon.

By a comparison of the deflections from the tangent to the orbit, produced by the attractive force of the sun, on the earth, and on Jupiter, the ratios are found to be : : 24·402 : 0·9019; but by the law of the inverse squares, they are found to be : : 24·402 : 0·9024—so closely do they agree. By considerations such as these the universality of the law of gravitation is established.

309. The ratio of the mass of the sun (S) to that of the earth (E) may be found as follows :—

In the case of the earth and moon we have seen that the earth's attraction deflects the moon from the tangent to her orbit 0·0536 of an inch in a second; and we find

in a similar way that the sun deflects the earth from the tangent 0·119 of an inch in a second. These numbers are in the proportion of 1 to 2·233. Let S be the mass of the sun, M of the moon, and E of the earth, D and d their distances, then $\frac{S}{D^2} : \frac{E}{d^2} : : 2{\cdot}233 : 1$, or $S = E \times \frac{D^2}{d^2} \times 2{\cdot}233$. If we assume that $D = 385d$ (Art. 132), then $S = E \times 331030$. If the larger distance of the sun be taken, $D = 400d$ nearly, and $S = E \times 355000$ nearly.

Having got the mass of the sun we can get the ratio of the mass (M) of any planet, accompanied by a satellite (mass m) to the sun; for if R and r be the distances of the planet from the sun, and of the satellite from the planet, and T and T′ their periodic times, $\mu S = \frac{4\pi^2 R^3}{T^2}$ and $\mu M = \frac{4\pi^2 r^3}{T'^2}$; consequently $S : M : : \frac{R^3}{T^2} : \frac{r^3}{T'^2}$. When planets are not accompanied by satellites we get their masses from their mutual perturbations.

Where this method is applied to the case of the sun and earth, it gives $\frac{S}{E} = \left(\frac{D}{d}\right)^3 \left(\frac{T'}{T}\right)^2$. Where T′ and T denotes the length of a month and a year respectively. For an approximate result we may assume $D = 385\ d$, and $T = 13\ T'$: substituting these numbers we get $\frac{S}{E} = \frac{57{,}066{,}625}{169} = 337{,}672$.

310. The mass of the moon is found from comparing the actions of the sun and moon upon the waters of the earth, in producing the tides, or from the lunar nutation (Article 90) which depends upon the moon's attraction alone, and consequently upon her mass. Having measured the amount of the effect, we can calculate the magnitude of the cause. Both methods agree in giving to the moon a mass of $\frac{1}{73}$ that of the Earth. The solar nutation is to the lunar nutation as 2 : 5, which is just the proportion to the solar tides to the lunar.

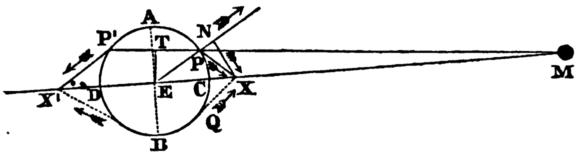
311. The following table gives the masses, densities, and specific gravities of the sun and the principal planets. The masses have been calculated by Leverrier:—

	Mass.	Density.	Spec. Grav.
Sun, ...	354,936	0.25	1.37
Mercury, . .	0.12	2.01	10.97
Venus, . . .	0.88	0.97	5.30
Earth, . . .	1	1	5.46
Mars, . . .	0.13	0.72	3.93
Jupiter, . . .	338.03	0.24	1.31
Suturn, . . .	101.06	0.13	0.71
Uranus, . . .	14.29	0.15	0.82
Neptune, . .	24.65	0.27	1.47

312. THE TIDES.—The daily ebb and flow of the tides, and the changes which take place in the height of the waters of the sea during a lunar month, belong rather to the class of terrestrial than to that of celestial phenomena. However, as they can be shown to be produced by the action of the sun and moon upon the earth and the waters which surround it, a sketch of the theory of the tides is not unnaturally expected in a treatise on astronomy. The full mathematical discussion of the theory of the tides is one of the most difficult questions of analysis, and it must be sought for in the works of Newton, Bernouilli, Laplace, or Airy. The scope and limits of the present work enable us merely to show, from elementary geometrical considerations, the manner in which the phenomena of the tides depend upon the attractions of the sun and moon.

Let us begin by considering the moon to be at rest, and the earth to have no rotation about its axis, and to be covered by an ocean of water of an uniform depth. Since the moon's attraction is exercised not only on the earth, but on the waters which surround it, and which are free to move over its surface, and since the attraction is greater in proportion as the square of the distance of the point attracted from the moon is diminished, the waters nearest to the moon will be more attracted than

the earth, and piled up in a heap under the moon; while the earth, being more attracted than the waters in the hemisphere away from the moon, will be drawn towards that body away from these waters, and they will, with respect to the earth, form a heap on the other side and away from the moon. The waters will in fact form a prolate spheroid, the greater axis of which will be directed towards the moon. In order to



estimate the force which acts upon the waters, let M (fig. 45), be the moon, E the earth, P a particle of water. If the line PM represent in quantity and direction the attraction of M upon P, and if X be taken on the line ME, so that $MX : MP :: MP^2 : ME^2$, XM will represent in quantity and direction the action of M upon E. Now let us reduce the earth to rest with respect to the moon, by applying to the earth, and to each particle of water on its surface, a force equal and opposite to XM, then the particle P will be acted upon by a force PM, and by another equal and parallel to MX, the resultant of which will be PX; it is clear then that for every particle of water in the hemisphere ACB, the action of the force of the moon upon the water will be towards some point in the line CM. If P′ be a particle of water on the opposite hemisphere, and if MP′ represent the moon's attraction upon it, and MX′ be taken so that $MX' : MP' :: MP'^2 : ME^2$, by a similar method of reasoning, it may be shown that the moon's distnrbing action upon every particle of water in the hemisphere ADB will be towards X′ away from M.

In order to estimate the magnitude of this disturbing force PX, which is to $\frac{M}{PM^2} :: PX : PM$, and consequent-

ly $= \frac{M}{PM^3} \times PX$, we resolve PX into PN, which lifts the water vertically, and NX, which moves the water horizontally towards C. Let ME $= a$, MP $= d$, PEC $= \theta$, EP = R, then since EMT is always considerably less than 1°, ME nearly = MT, and EPT nearly = PEC; but by the proportion already given, we have MX $= \frac{MP^3}{ME^2} = \frac{d^3}{a^2}$, $\therefore$ EX = EM − MX $= a - \frac{d^3}{a^2} = \frac{a^3 - d^3}{a^2} = (a-d)\left(1 + \frac{d}{a} + \frac{d^2}{a^2}\right) = 3\,(a-d)$, $\left(\text{since } \frac{d}{a} \text{ nearly} = 1, \text{ being at its least } \frac{59}{60}\right)$ = 3TP = 3R cos θ. Again PN = EN − EP = EX cos θ − EP = 3R cos² θ − R, $\therefore$ the force PN $= (3R\cos^2\theta - R), \times \frac{M}{d^3}, = \frac{1}{2} R\,(1 + 3\cos 2\theta)\,\frac{M}{d^3}$, and NX = EX sin θ = 3 R cos θ sin θ, $\therefore$ the force NX $= \frac{3}{2} R \sin 2\theta \times \frac{M}{d^3}$.

The force PN vanishes when $\cos^2\theta = \frac{1}{3}$ or $\theta = 54° \, 44'$; from A to that point it is negative and depresses the water, and thence to C positive and elevates it.

313. The action of the sun upon the water would produce a similar effect, though less in degree, since Newton has shown that the proportion of the moon's disturbing force to that of the sun is somewhat less than $2\frac{1}{2} : 1$.

If the earth did not revolve on its axis, and the sun and moon were stationary, and in the same or in opposite parts of the ecliptic, the greater axes of the two ellipsoids formed by the sun and moon's attraction would coincide, and both would conspire to raise the water under the moon; or the greater axis of the ellipsoid pointing to the moon would be the greatest possible. If the lines drawn to the sun and moon from the earth were at right angles, the elevation arising from the moon's attraction would be counteracted in part by the depression arising from the sun's, and the axis of the com-

posite ellipsoid pointing to the moon would then be the least.

314. This is called the equilibrium theory, and would give high water always under the moon, and spring tides when the moon is in conjunction or opposition, and neap tides when the moon is in quadratures. But there are two suppositions involved in it which modify considerably the results, and which are different from the actual conditions of the question.—1st. The earth is not at rest, but is revolving round an axis carrying the water along with it in its diurnal motion. 2nd. The earth is not uniformly covered with water, but there are oceans and seas of variable depth, and large intervening continents which prevent the formation in full of this ellipsoid of water. The sun and moon also change their position in the heavens with respect to the earth, and to each other, from day to day, so that there is never time for the spheroid to be actually formed. Consequently, this theory must be considered only approximate, and it can be shown that so far from high water being under the moon, as the equilibrium theory would require, it is low water under the moon and high water at the points A and B, 90° distant on the earth's surface from the point C.

315. The following demonstration of this has been given by Mr. Abbott.[a] Let the moon be supposed to be fixed, and the earth to revolve in the direction of DACB, carrying the ocean along with it. The tangential force NX alone is to be considered, as the normal force PN goes to diminish or increase gravity by a very small amount. In the course of a lunar day every particle of the ocean is subjected to precisely the same forces acting in the same order of succession and for the same periods, being accelerated from A to C, then retarded from C to D, accelerated from B to D, and retarded from D to A.

This being so, it is obvious that the particles of water will be moving faster which have been for a long time acted on by an accelerating force, and the velocity will be a maximum when the accelerating force has acted during its full period—viz., through one quadrant. On

[a] Philosophical Magazine, Jan. 1870, Feb. 1871.

the other hand, those particles will be moving slower which have been longer acted upon by a retarding force, and the absolute velocity will be a minimum when the retarding force has acted during its full period, or through one quadrant. The maximum velocity is therefore at D and C, the minimum at A and B.

It is also clear that the tide will be rising where each portion of water is moving faster than that just in advance of it, or, in other words, is flowing in faster than it flows out. Where this is continued for the maximum time the tide is highest. On the other hand the tide will be falling where the water is moving slower than that in advance of it—*i.e.* is flowing out faster than it flows in. Where this has continued for the maximum time the tide is lowest.

Now consider the point P in the quadrant AC. The water now passing P has been subject to an accelerating tangential force NX during the whole time since it has passed A; longer, therefore, than any particles behind it between A and P. It is, therefore, moving faster, and as the water in the space between P and any point behind it is flowing out faster than it is flowing in, the tide is falling. This is the case through the whole quadrant AC.

At C the tangential force changes its direction and becomes a retarding force; a particle at Q has been subject to this retarding force longer than one behind it between C and Q, and is therefore moving slower. Here, therefore, water is flowing in faster than it flows out, and the tide is rising; this holds through the quadrant CB. Similarly it may be shown that the tide is falling all through BD and rising all through BA. Hence it is highest at A and B, lowest at C and D. It falls fastest when the difference between the velocity of P and that of a particle behind it is the greatest—that is, when the force on P is a maximum, or $\frac{3\,M\times R}{2\,d^3}\sin 2\,\theta$ is a max., or when $\theta = 45°$, since d does not change much; similarly it rises fastest at a point half way between C and B.

On the whole, then, in an equatorial canal the water assumes the form of an ellipse, and as it is the earth

which is rotating, this ellipse does not change its absolute position, except with the moon's monthly motion. Relatively to the earth it is moving westward at A and B, eastward at C and D. Taking friction into account, we can see that the effect of friction is to accelerate the time of high and low water, for as the water approaches C the tangential force diminishes gradually until it vanishes at C; therefore, at some point a little before C the tangential force must have been equal to the force of friction, and destroyed by it, after which friction acting from C to A is greater than the tangential force towards C; the effect of friction then is to produce low water a little before C in the direction of CA. Approaching B the ocean is moving slower than the earth; therefore, here friction tends to accelerate it, while the retarding force is decreasing to zero, the two forces then must be equal and opposite at a point near B between B and C, after which the velocity again increases. It is high water consequently at that point.

Mr. Abbott shows, moreover, that there is a balance of lagging, or in other words, a continuous current, relatively to the earth, westward; and that friction being considered, the effect of this is to retard the earth's rotation, or to lengthen the day.

316. M. Delaunay had been led to this conclusion; and Mr. Airy, by extending the mathematical approximations, has to a certain extent confirmed it. The actual retardation of the earth's diurnal motion cannot be deduced from mathematical investigations, owing to the existence of great and irregularly-shaped continents on the earth's surface. All that can be done is to ascertain the moon's acceleration, as determined by considering very ancient eclipses, and by comparing them with the lunar tables calculated on the principles of gravitation alone, supposing the earth's rotation to be unchanged. This is a problem for future astronomers to investigate. In consequence of friction of the water against the bed of the sea, the narrowness of channels, and other local circumstances, the high water is retarded at each place by a certain time, constant for each port, which is called the *Establishment of the Port.*

CHAPTER XIX.

THE RECENT DISCOVERIES IN REGARD TO THE PHYSICAL CONSTITUTION OF THE HEAVENLY BODIES—SOLAR SPOTS—SPECTRUM ANALYSIS.

317. THE study of the physical properties of the heavenly bodies has latterly received a new impulse. The distance of these bodies—even those which belong to our solar system—is so immense that the most powerful telescopes can show us but little on their surface. Even in the case of our nearest neighbour, the moon, the telescope admits only of a very summary survey, and though, it is true, her general topography in every part of the visible surface is better known to us than that of many regions of the earth, yet all the minor details remain invisible, as no object can be distinguished whose diameter is much less than one mile. On the sun a line equal to 410 miles appears to us as a mere point, the angle subtended by it being one second. Thus, our inability to perceive anything, except the largest features, even through the most powerful telescopes, puts us to great disadvantage in the study of the physical nature of the heavenly bodies. Latterly, however, the application of the spectroscope to this study has opened an entirely new field for investigation, allowing us a deeper insight into the very nature of even the remotest bodies. It is thus that the opinions which had hitherto been held in regard to these bodies have been greatly modified during the last few years.

318. The sun, when viewed through a telescope, appears dotted with small light and dark points, and with ridges of different shades. The light points, or granules,

which are somewhat of the shape of rice grains, are about 1″ long, rarely 2 or 3 seconds. They are seen closely united in groups, and the greater or less degree of closeness creates a difference in the brightness of adjoining areas which produces the coarse mottling of the sun's surface; and the effect of this is increased by the difference of brightness of the material which fills the intervals of these groups. The contrast of the brightness of these granules and the surrounding matter is so great, that the granules really are the source of nearly the whole light which the sun emits. The extreme mobility of these granules has suggested to all observers that they are of the nature of incandescent clouds floating in a gaseous and comparatively non-luminous medium. The most remarkable phenomena observed on the surface of the sun are, however, the solar spots. They are of various sizes, from very small spots to those whose diameter is ten times the diameter of the earth. The smaller ones appear regular, round, and black; but the larger ones are more complicated, being surrounded by a more or less concentric ring, called the penumbra, consisting of matter brighter than the dark portion (nucleus), but considerably less luminous than the general surface of the sun. The dark nucleus, however, is in reality by no means non-luminous, as we easily perceive when at the time of a transit of Mercury or Venus we are able to compare the nucleus with the really black disc of the planet. Near the edges of spots the granules change their character and become enlarged. Their length exceeds then 3 or 4 times their breadth, and they look very much like willow leaves placed round a hole, so that all the larger axes are in the direction of the radius of the umbra. In the penumbra of large spots there are usually also fine brilliant stripes, which Mr. Dawes compared to blades of straw, the shaded appearance of the penumbra being produced by the intervals between these stripes, which are covered with a fine veil moderately luminous. There are, however, spots without such stripes, when the penumbra is formed merely by such a thin veil. Sometimes the willow leaves or the

stripes are changed into such veils, and the forms of the spots are often so changeable that they alter their appearance even during the time of observation. It is as if portions of the luminous matter are continually breaking away and drifting on to the umbra where they seem to dissolve and disappear. They must thus either be cooled and become more luminous, or they may become more heated and return to the gaseous state, just as clouds in our atmosphere are dissolved by an elevation of the temperature, and are changed into transparent vapours.

When the spots are near the centre of the disc, their borders differ little in brilliancy from the general surface. But when the spots are near the limb, their edges appear surrounded by bright spots which are called faculæ. These remain visible up to the very time when the spots disappear, and seem to be upheavings of luminous matter. They are always seen in close proximity to the spots, and generally precede their appearance.

The spots are not seen all over the disc of the sun, but are confined to a zone of 35° on each side of the equator, though sometimes, but very rarely, one is seen beyond those limits. But they are never seen in the neighbourhood of the pole, and very rarely also in the immediate neighbourhood of the equator within 3 degrees on either side. They have a great tendency towards being disposed in groups parallel to the sun's equator. The number of spots visible at different times varies very much, and their greater or less frequency follows a certain law, so that the time of maximum and minimum of spots may be predicted with great accuracy. By the investigations of Mr. Schwabe and Professor Wolff, of Zurich, it has been ascertained that this period is about 11·2 years ; varying, however, sometimes several years, as if there existed minor periods, such as we find in the perturbations of the planets. This period, in its duration as well as in its different phases, seems to follow closely the period of the terrestrial magnetism.

The motions which the spots show in the direction from east to west have, since the time of their first dis-

covery, been used to ascertain the time of rotation of the sun. Modern investigations have revealed the fact that the spots near the equator move the fastest, and that the angular velocity decreases regularly with the latitude of the spot, the period of rotation of a spot on the equator being $25^d.2$, while that of a spot in latitude 45° is $21^d.8$. They must thus have motions of their own, which, however, are not quite regular, the above numbers giving only the average velocity. The character of the various spots is very different. Sometimes they are seen only for a short time, while at other times we see them perform several entire revolutions, Mr. Schwabe having observed once a group of spots during eight such revolutions. Sometimes they are very changeable, while at other times they are more constant, and small and regular spots usually preserve their outline for a long time. But the majority show a peculiar change according to their position on the sun's disc, which was first pointed out in 1773 by Professor Wilson, of Glasgow. When a spot is carried by the rotation of the sun from the centre of the disc toward the limb, the umbra will always appear to encroach upon that side of the penumbra which is nearest the centre of the disc. While the umbra and penumbra of a regular spot, when near the centre, appear as two concentric circles, they will be converted into ellipses when the spot is near the limb, but the ellipses will not be concentric, the centre of the umbra being nearer the centre of the disc. By degrees the umbra becomes partially hidden by the penumbra and disappears altogether in the neighbourhood of the limb, the penumbra alone remaining visible. This change of form is always observed in the case of a regular round spot, but whenever a spot undergoes considerable real changes while approaching the limb, these may compensate the other; but among a large number of spots we always find a majority in favour of this law of Wilson. Deviations from this rule are most frequently to be attributed to the disturbing effect of neighbouring spots, as they occur mostly in groups of spots, while the regular and

insulated spots behave pretty generally in accordance to the rule.

319. This change of the appearance of solar spots depending upon their position on the limb is evidently due to perspective, showing that the umbra must be nearer the centre of the sun than the penumbra. Wilson, therefore, immediately after this discovery, proposed the theory that the sun consists of a non-luminous dark body, covered with a light stratum of an incandescent nature, which is the source of all light and heat. The spots are thus openings in this luminous stratum, made by an elastic fluid, generated in the dark body of the sun. Sir W. Herschel adopted the same hypothesis, only with this modification, that he interposed immediately below the luminous stratum a non-luminous atmosphere, which reflected nearly all the light received from above, and at the same time served to explain the penumbra. This hypothesis had been until lately generally adopted, as it explained the principal features of the solar spots, and had the additional charm of rendering the body of the sun inhabitable, and perhaps by beings not differing much from ourselves It was, however, lately overthrown by the brilliant discoveries of Kirchhoff.

320. About the year 1815 the celebrated German optician Fraunhofer discovered that the spectrum of the sun, produced by refraction of its light through a prism, is interspersed with black lines which always retain the same invariable position with respect to each other. In order to observe them, we let the light of the sun fall through a very narrow slit with parallel edges upon a prism, and observe the spectrum through a little telescope. Such an apparatus is called a Spectroscope. It is a little modified by the addition of a collimating lens between the slit and the prism, and whose focus coincides with the slit, so that the rays leave this lens parallel as if coming from an infinite distance, and instead of one prism several may be used in order to increase the dispersion. As the white light is decomposed by the prism on account of the difference of refraction with which light of different colours is endowed, this dis-

covery showed that rays of a certain refrangibility are deficient in the solar light. The cause of this remained, however, unknown, only Sir David Brewster showed that the light loses some of its rays by the absorption of the atmosphere of the earth, as he found that new black lines appear in the solar spectrum when the sun approaches the horizon, and he suggested, therefore, that the invariable lines which show themselves in the solar spectrum may be owing to an absorption which takes place in the atmosphere of the sun. The mystery was at last explained by the study of artificial flames. The spectra of simple incandescent bodies, like that of a piece of chalk made incandescent by the flames of oxy-hydrogen gas, or that of the incandescent pieces of carbon in a common flame, are continuous, and show no dark lines. The spectra of incandescent gases, for instance those arising from the combustion of metals, are discontinuous, and consist of bright lines of special colours. This discovery contained the germ of a new method of chemical analysis. It was found that the spectrum of each metal has bright lines of special colours which retain an invariable position, and may be seen in the spectrum of an electric spark whose electrodes consist of that metal, or are covered with a solution of any of the salts, as well as when such salts are mixed with a flame. Thus any flame containing sodium furnishes a spectrum in which a yellow line of extraordinary brightness is depicted on the general ground of the usual colours. It had been remarked that this sodium line corresponds to the black line in the yellow part of the solar spectrum, which had been named D by Fraunhofer, who denominated all the principal lines by letters of the alphabet. In order to prove this thoroughly, Kirchhoff produced a faint solar spectrum, and then placed a sodium flame before the apparatus. Immediately the black line D was changed into a bright line, and he showed that this line is only the bright line of sodium extinguished or reversed. When the light of an incandescent piece of chalk, which has no dark lines, was made to pass through the spectrum of the sodium flame, the bright line was con-

verted into a dark one. It was thus shown that the sodium flame absorbs the rays of the same refrangibility as those which it emits, and is transparent for all other rays. The spectrum of any incandescent gas must thus be reversed—that is, the bright lines must be converted into dark ones, if rays of another source of light, which is sufficiently bright, pass through it. Kirchhoff then showed that there were lines in the spectrum of the sun which exactly correspond to the iron lines and those of other metals, and it is therefore necessary that the rays which form the solar spectrum have passed through vapours of iron and those other metals which reversed those certain rays; and as this absorption cannot take place in the atmosphere of the earth, as in that case the lines would change with the altitude of the sun, these vapours must be present in the atmosphere of the sun. We must assume, then, that there is a luminous stratum, the photosphere, which emits light of every refrangibility, and which alone would give a continuous spectrum, and that this is surrounded by an atmosphere of a lower temperature, but still high enough to sustain all the various metals, as iron, sodium, &c., as vapours.

321. At the time of a total eclipse of the sun very strange phenomena are seen, to which the attention of astronomers was first called at the eclipse in 1842. When the last ray of the sun disappears the moon appears surrounded by a ring of silvery white light, about equal in breadth to its diameter, which is called the corona. There is always a zone of very vivid brightness, 3 to 4 minutes wide, round which there is another zone whose light very rapidly diminishes with the distance from the centre. From the first zone usually start a number of bright rays, whose length varies, and attains sometimes double the diameter of the moon. The corona is always concentric to the sun, as it is brightest in that part where the moon is nearest to the limb of the sun, and it has been established that at least the portion nearest the sun belongs to the sun. But the most remarkable phenomena of an eclipse are the protuberances. At the beginning of the eclipse, we see on that side of the moon

where the last ray of light has disappeared, luminous prominences, usually of a light rose colour, and of various forms; sometimes they are conical, mountain like, or curved like horns, and sometimes they are even entirely detached from the body of the sun, like clouds. As the eclipse advances they grow smaller, as if being covered by the moon; while on the opposite side similar prominences make their appearance, which gradually increase in size till the end of the total eclipse, when they rapidly vanish after the first ray of sunlight reappears. The observations of the eclipse of 1851, and especially those of the eclipse of 1860, which was observed in Spain, had established beyond doubt that these prominences are real phenomena belonging to the sun, and therefore, when Kirchhoff had made his brilliant discoveries, the next total eclipse, which happened in 1868, was expected with great anxiety, in order to examine those prominences with the spectroscope, and thus to discover their nature. It was then found that the spectrum of those protuberances is discontinuous, and consists of a small number of bright lines, all of which, with the exception of a yellow line close to the sodium line, are the hydrogen lines; thus showing that these protuberances are chiefly composed of upheavings of incandescent hydrogen gas, a stratum of which surrounds the whole sun.

Mr. Janssen, who observed the eclipse at Guntour, was so struck with the brilliancy of their rays that he immediately conceived the idea of observing them even in the full light of the sun; for the spectroscope disperses and enfeebles the light of the sun, while the bright rays retain their brightness. On the day following the eclipse, he thus succeeded in seeing the bright lines of a protuberance, and on the day when the news of this discovery reached Europe, Mr. Lockyer had independently discovered these lines in full daylight. Indeed, he had foreseen the probability of this, in case the spectrum should consist of bright lines, and had a powerful spectroscope constructed for examining the borders of the sun for that purpose, and had the pleasure of seeing the bright lines on the first day he was able to use the instrument.

The spectrum of the corona has not yet been satisfactorily settled, and the eclipse of December, 1871, is looked for with great anxiety, and great preparations are being made for it, to settle this question, and to investigate more fully the nature of the corona.

322. The dark lines in the spectrum of the sun are not the same all over the disc. Some fine lines, which are barely visible near the centre, become very plain near the limb. In the neighbourhood of spots, and especially of faculæ, the hydrogen rays become more feeble and sometimes disappear altogether, and become even reversed, owing to protuberances which, as we have seen, consist especially of hydrogen gas. In the interior of spots the spectrum is greatly modified. Some lines become extremely black and large; others become indistinct; while many undergo no change at all. This must be owing to a special absorption produced by some substances in the interior of the spots. They are, therefore, regions where the absorption is stronger, especially that of some metallic vapours, which, owing to their density, occupy the lowest parts of the irregularities existing in the photosphere. They may be cavities produced by eruptions of gas from the interior, especially as the protuberances, and probably the faculæ, must own their existence to such eruptions.

323. The discoveries made by the spectroscope, in regard to the constitution of the sun, agree very well with a hypothesis proposed by Kant, and adopted by Laplace, which is known as the nebular theory.

On account of certain properties common to all members of the solar system, such as their common motion from west to east, their rotation in the same direction, as well as the small inclination of their orbits with respect to the sun's equator, he was led to attribute the formation of our planetary system to the gradual condensation and breaking up of a single nebula, in which the cosmical matter at present condensed in the sun, the planets, and the satellites, was primitively diffused through the entire space which this system occupies. If we assume those masses spread only as far as the

orbit of Neptune, it would be as rarified as the air under our best air pumps. Now, if such a mass condenses, precipitating itself to a central point, an immense amount of heat must be developed, which in the case of the sun has been calculated to be millions of degrees. All the bodies of the solar system have been thus originally in the same state of high temperature, and differ, in their present state, in consequence of their different masses having reached different stages in the process of cooling, to which they are subjected by the continual radiation of heat into the surrounding space. We know that the earth was once of a high temperature and that the interior is still so, the surface alone having cooled down.

But the sun is still in a state of incandescence, so that his atmosphere contains still those substances, as gases, which on the earth have long since been condensed and fixed as solid rocks. Even the interior of the sun may still be in a gaseous state, which, however, on account of the enormous pressure, must be very different from any such state which we observe on the earth. Owing to the high temperature the different substances must be in a state of entire dissociation—that is, are in presence of each other, without being able to combine. This combination can only go on on the surface, when the radiation has lowered the temperature. Precipitations will be formed and clouds of incandescent fluid or solid particles, whose radiation of light will be much greater than that of the mass of the interior, and, therefore, will form a real photosphere. These particles sink again, on account of their gravity, into the lower strata until they meet a temperature by which they are converted to their primitive state, while other masses of gas come to the surface and go through the same process. The phenomenon which we observe on the surface of the sun would thus be owing to the process of cooling, which, in the course of ages, would at last produce a state in which the sun would cease to have the power of supporting life on the surface of the planets.

324. The spectra of the moon and of the planets would be of the same nature with, and show the same lines as the solar spectrum, since these bodies shine merely by reflected solar light. Only, as this reflected light must pass twice through the height of the planetary atmosphere before it reaches our eye, the spectra may show some new lines owing to absorption in these atmospheres, and thus give us an insight into their nature. Now, the moon shows no other lines than those visible in the solar spectrum, where the sun has a considerable altitude. In regard to it, therefore, the result is wholly negative as to the existence of an atmosphere, and thus the spectroscope confirms the opinion which astronomers had derived from other facts. Mr. Huggins came to the same negative result in observing the occultation of a star with a spectroscope. Watching the spectrum of the star while the dark limb of the moon was closely approaching it, and just before it was disappearing, he did not perceive any alteration of the spectrum which might be attributed to a lunar atmosphere.

The spectrum of Venus shows that the atmosphere of this planet is somewhat similar to that of the earth. The spectrum of Mars shows a few strong lines in the red, and near the line F there is a series of strong bands which continue towards the more refrangible end of the spectrum. In the spectrum of Jupiter a very strong line is seen, besides some others which are owing to his atmosphere. It is, however, coincident with a faint band in the sky spectrum, and shows the presence of similar vapours, probably aqueous vapours, in our own atmosphere; but as another line does not correspond to any of the air lines, his atmosphere must contain also some other gas which is not a constituent part of our atmosphere. The small density of this planet makes it probable that it is not yet in a solid state; the spots and bands seen on its surface are always very variable, and its atmosphere seems still to be in a state of revolutions analogous to those through which the earth passed at past geological epochs. The same is the case with Saturn, whose spectrum is also similar to that of Jupiter. The

spectrum of Uranus shows six strong bands, owing to the absorption of its atmosphere, one of which seems to coincide with the line F in the solar spectrum, one of the hydrogen lines.

The spectra of the few comets which have so far been examined are discontinuous, and consist usually of some green, yellow, and red bands. According to some observations, the spectrum of the coma is continuous, probably produced by reflected sunlight, while that of the nucleus shows several brilliant rays, emanating from a gas in a state of incandescence.

325. The spectra of the fixed stars may be divided into different classes. First, we have the stars which are generally called white, like Sirius, Wega, which show a continuous spectrum, interrupted by four black lines which correspond to the four bright lines of hydrogen when at a high temperature. The brightest stars of this class show, besides, a line in the yellow which seems to coincide with that of sodium, and some lines in the green. The second class are the yellow stars, like Arcturus, Aldebaran, whose spectra very much resemble that of the sun, being intersected by fire lines which coincide, more or less, with those in the solar spectrum. These stars must therefore be, in a physical state, very similar to that of our sun. The third class, which is not very numerous, contains stars whose spectra are interrupted by broad dark bands. These stars are red or orange-coloured, and are mostly all variable, and the bands are also variable, according to the brightness of the stars. The spectrum of the nucleus of solar spots shows, as we have seen, stronger lines of absorption, and if, therefore, our sun was deprived of its brilliant photosphere, it would present a spectrum very similar to these stars. Besides these classes of stars there are a few which contain the bright lines F and C of hydrogen, and besides a brilliant ray in the yellow, which seems to coincide with a similar ray in the spectrum of the protuberances of the sun.

326. The spectrum of some of the variable stars changes, as we have seen, with the brightness of the stars,

owing to greater or lesser absorptions. Thus, as the variable star in Cetus grows brighter, the dark bands in yellow and green seem to become less distinct and less black. We see thus that the variability of these stars may be caused by phenomena in connexion with the cooling process which they undergo, and of which we spoke before in reference to the sun. The play of forces between the cooling surface and the interior will be the more efficient, the more free is the communication between the interior mass and the surface—that is, as long as the primitive gaseous state is not entirely changed, there will be a thin stratum of a photosphere, when the chemical processes are going on with great regularity. But a time will come, when the free interchange between the interior and the surface will be disturbed, and the upward currents will meet with greater obstacles; there can then be abnormal distributions of density and temperature, and a state of unstable equilibrium will exist, which will be changed suddenly to reproduce the primitive state. It is natural that the same phenomenon will return after certain intervals. At first, the periods will be quite regular and little perceptible, but by degrees they will become more distinct, in proportion as the photosphere increases in density, and the descending currents go to greater depth. The changes will grow very irregular, until finally they become only exceptional and spasmodic. Then suddenly a large mass of high temperature will be thrown up to the surface, and thus a sudden increase of the brilliancy of the star will be produced which will gradually diminish. These phenomena are the precursors of final extinction, and are observed only in faint stars, which suddenly, and only for a short time, increase to a great brightness, and are then mistaken for new stars. The sun at present is in that state when the changes are only slight and tolerably regular. But among the stars we find all the different stages represented at the present time from such stars, where the formation of a photosphere has advanced only so far that we see either no change, or only slight and periodical changes, to such, where they are sudden and spasmodic.

Such are the temporary stars like the one which suddenly appeared in 1866 in Corona. While this star was in the state of greatest brilliancy it consisted of two different spectra superposed one upon the other. The main spectrum was like that of the sun, formed by light coming from an incandescent photosphere, and having undergone absorption by surrounding gaseous matter. But above this was another, consisting of a small number of bright lines such as are produced by incandescent gases. Two of the bright lines coincided very nearly with the hydrogen lines C and F, and were produced perhaps by this gas. In twelve days the star decreased again to the eighth magnitude. This shows that the sudden increase of brightness must have been owing to an eruption of a highly heated gas from the interior through the photosphere, which before this had nearly ceased to be luminous.

There are, however, variable stars, where the spectroscope shows no change in the spectrum during the increase and decrease of the light. Such is, for instance, the star Algol. The variations in the light of such stars cannot, therefore, be attributed to a more or less strong absorption, or to more or less developed spots, but must probably be explained by the periodical interposition of opaque bodies moving about them.

327. While the spectra of all stars show absorption lines, the nebulæ have either a continuous spectrum, or one consisting of a few bright lines. Of the first class of nebulæ which show a continuous spectrum, a large proportion is resolvable into stars—and it is very probable that they all are clusters of stars—which appear only as nebulæ on account of their great distance, or on account of the intrinsic faintness of the stars, which does not allow us to see them separate, with our present means of observation. These spectra may not be without absorption lines, but they may be only very difficult to be seen, and the fact that some of the spectra appear irregularly bright in some points speaks in favour of their existence.

As to the other class of nebulæ which show spectra of bright lines, we must infer that they consist of masses

of incandescent gases, a fact which explains the feeble light which these bodies emit. They were formerly believed to be at much greater distances from us than the fixed stars, on account of their faintness. But this is no reason at all, if we consider the immense difference in the radiating power of incandescent gases and solid bodies, or gases in which such incandescent solid or fluid particles are floating. In arguing to their distance from their light, we should thus commit a still greater fallacy than if we should assume the distances of the fixed stars to be in the inverse ratio of their brightness.

The bright lines which these nebulæ show—for instance, the celebrated one in Orion—are usually three in number. One of these coincides with the hydrogen line F, the brightest one coincides with the brightest of the nitrogen lines, while the third corresponds to no line of the known elements. These nebulæ may still be in a state of dissociation, the high temperature preventing as yet any chemical action which would produce the various elements from this primitive matter. But if there is a gradual development of nebulous matter, it is surprising that we do not see some of these nebulæ in a more, others in a less advanced state; showing, by the greater or lesser number of lines, the presence of a greater or lesser number of substances. It is true that the well-known ring nebula in Lyra and the Dumb Bell nebula show a spectrum with only one bright line, while a nebula in Aquarius shows four; but it would seem as if such cases ought to appear in greater number and greater variety, such as we see in their forms and structure. It may be, therefore, that these gaseous nebulæ are of a different order of cosmical bodies, distinct from that of the sun and the fixed stars.

CHAPTER XX.

PROPER MOTIONS OF THE STARS—MOTION OF THE SOLAR SYSTEM—DISTANCES OF THE STARS—ANNUAL PARALLAX—BINARY STARS—DISTRIBUTION OF STARS.

328. The places of the fixed stars are given in catalogues according to their right ascensions and declinations. Now these change, as we have seen, owing to the precession of the equinoxes and nutation. These changes do not, however, affect the relative positions of the stars, but merely the fundamental plane to which we refer their places in the heavens. The plane of the earth's equator, in fact, slowly changes its position in respect to an absolutely fixed plane, and so the co-ordinates of the star are changed as the axes of reference are altered. But if we compare very distant observations of the fixed stars we find that the changes in their right ascensions and declinations are not entirely explained, even allowing for errors of observation, by this change of the equator. In fact, if we proceed to determine the amount of the precession of the equinoxes from different stars, we find considerably different values, which shows that besides this apparent change in the positions of the stars, owing to the displacement of the fundamental plane to which they are referred, they have real motions of their own, in consequence of which they change their places relative to each other. In fact the amount of precession can be ascertained only from the mean given by a large number of stars on the supposition that in this mean the real motions of the stars, as they proceed in different directions, will destroy each other. Thus the value of the precession generally

adopted was deduced by Bessel from more than 2000 stars whose places, determined by Bradley in the middle of the last century, he compared with his own observations made at Kœnigsberg. The difference of the change of the right ascension and declination of a star from that which it ought to show according to this value of the precession is taken as the real proper motion of the star in space as projected upon the celestial sphere. For some stars this motion is very considerable, but it bears no relation whatsoever to the brightness of the star; some of the smaller stars showing the largest proper motions, as, for instance, a small star of the tenth magnitude between the Great Bear and Canes Venatici (No. 1830 of Mr. Groombridge's Catalogue). This star has a proper motion of 7″ of arc annually. The double star, 61 Cygni, moves 5″ annually, the two stars composing it being carried along in parallel lines with a common velocity. The bright star Arcturus has an annual progressive motion of 2″. Although these motions are at first sight very irregular, Sir W. Herschel, in 1783, led by a consideration of those which had been at that time discovered, came to the conclusion that, on the whole, the stars seemed to move in the direction from a point near the star λ Hercules, and he concluded that these motions are, partly at least, merely apparent, and owing to a proper motion of the sun carrying with it the entire planetary system, and moving in the direction of that point. For if the sun had such a motion, every star must seem to move away from such a point in the great circle passing through the star and the point to which the sun is moving. Since that time the proper motions of the stars have been more extensively and accurately observed, especially by Argelander, and calculations of the motions of the sun which they indicate have been made by this astronomer and others; and the mean of all these estimates gives a point which does not differ much from that originally assigned by Sir W. Herschel. The proper motions of the stars in the southern hemisphere have likewise been subjected to similar calcula-

tions, and have led to a very similar result as to the direction of the solar motion. There can then be little doubt that the sun is really moving in space towards a point whose right ascension and declination are 260° and + 30°.

The proper motions of the stars are thus partly due to this cause, but, besides this, they may have, like the sun, motions of their own, and the proper motions actually observed are the resultants of these two motions.

Although we can tell exactly in what direction a star must seem to move, owing to the motion of the sun, yet even though we should know the amount of this motion, we would not be able to calculate the amount of that of the star, unless we knew its distance from the sun. This being unknown we are unable to calculate the amount of the sun's motion in space from the proper motion of the stars.

Mr. O. Struve, by assuming hypothetical values of the relative distances of the stars of different magnitudes which had been deduced from the number of stars in the different classes, found that the average displacement of the stars requires that the motion of the sun should be such that if its direction were at right angles to a visual ray, drawn from a star of the first magnitude at average distance, its apparent annual motion would be $0''\cdot34$, a result which must, however, be considered only an approximation.

329. Distances of the Stars.—About the distance of the stars little is known at present, beyond the fact that they are so great that the diameter of the earth's orbit is a mere point compared to it. If we imagine a line passing through a star to move along the circumference of the earth's orbit, it will represent in succession the different directions in which the star is seen in the course of the year from different positions of the earth. This line describes the surface of a cone, and when produced beyond the star, will intersect the celestial sphere in a curve in which the star in the course of a year would appear to move. If the star is at the pole of the ecliptic, the axis

of the cone would be perpendicular to the orbit of the earth; its intersection with the celestial sphere, or rather with a tangent plane at the pole of the ecliptic, would be a circle. When the star is in the plane of the ecliptic, the cone would be changed into a triangle, and the star would move in the course of a year along a straight line. In general, however, the cone would be oblique to the plane of the ecliptic, and would intersect the celestial sphere in an ellipse, whose major axis would be equal to the angle which the diameter of the earth's orbit subtends at the star, while the minor axis would be shortened in the ratio of the sine of the inclination of the visual line of the star to the plane of the ecliptic or the sine of its latitude. Half the major axis of this ellipse expressed as an arc of a great circle—that is, the angle which the semidiameter of the earth's orbit subtends at that star—is called the annual parallax of the star. If this should be known, it gives immediately the distance of the star expressed in semidiameters of the earth's orbit (Art. 88). Now, this distance for the majority of stars is so great that this angle is no perceptible quantity, and the cone of the visual rays is virtually a cylinder, and the star at an infinite distance. So far only in the case of a few stars a parallactic change has been discovered with any certainty. We have seen (Art. 89), that for one star (α Centauri in the southern hemisphere) the parallax has been found to amount to nearly one second, all the others showing only a parallax of less than half a second. Now, a parallax of 1″ corresponds to a distance of 206,265 times the distance of the sun from the earth, and this we must therefore suppose to be the minor limit of the distance of the stars, a limit which is nearly twenty billions of miles. Light which moves, as we have seen, at the rate of 192,000 miles per second, would take 3,235 years, or three years and eighty-five days to move from such a star to the earth, and in general, if π be the parallax expressed in seconds or decimals of a second, the time in which light moves from the star to the earth is $\frac{3{\cdot}235}{\pi}$.

330. MODE OF DETERMINING ANNUAL PARALLAX.—We will say now a few words in regard to the determination of this parallax. It is easily seen, that when the longitude of the sun is the same as that of the star, the star, when in the northern hemisphere, will in its apparent ellipse be at the southern extremity of the minor axis, and when the longitude of the sun exceeds that of the star by 180°, at the northern extremity of the minor axis; again, when the longitude of the sun exceeds that of the star by 90°, the apparent place of the star will be at the eastern extremity of the major axis of the ellipse, and at the western extremity, when it exceeds that of the star by 270°. The apparent longitude and latitude have thus a maximum and minimum, both being separated by half a year; and those of the longitude occurring when the latitude has its mean value, and *vice versa.* At any other time the longitude and latitude correspond always to the projection of the point occupied by the star in its apparent ellipse upon the ecliptic and the circle of latitude.* Thus assuming the parallax of a star to be π, we can easily compute the effect of such a parallax upon the position of a star for any given day, or for any longitude of the sun; that is, we can compute co-efficients a and b, so that $a\pi$ and $b\pi$ would be the effect of the parallax upon the longitude and latitude; or that, if L and B are the mean longitude and latitude of the star, $L + a\pi$ and $B + b\pi$ would be the apparent longitudes and latitudes which the star would have, if its parallax was equal to π. Likewise also, as we know the angle between the circle of latitude and the circle of declination, we can compute similar co-efficients a' and b', so that if A and D are the mean right ascension and declination, $A + a'\pi$ and $D + b'\pi$ would give the apparent right ascension and

* It may be well to observe here that, as we have seen before, the stars describe, on account of aberration, similar ellipses whose major axes are 40″, but the places which the stars occupy in these ellipses are different from those which they would occupy if they had a parallax of 20″. The maxima and minima of *aberration* for any co-ordinate occur just where on account of *parallax* the star would be in its mean position

declination which the star would have if its parallax was π. Now, if we should observe the right ascension or the declination of a star during a year, and we should notice a periodical change in these, we can try whether they can all be represented within the limits of the errors of observation, by substituting in the formula $A + a' \pi$ in the case of a right ascension, or $D + b' \pi$ in the case of a declination, a certain numerical value for π. This operation requires the application of the Calculus of Probability, which at the same time supplies the most plausible value of π which that series of observations leads to. If there exists for π a positive value satisfying the observations within the limits of those errors to which they may be supposed to be subject, we take this as the value of the parallax. If right ascension, as well as declination, should give values in harmony with each other, this would be still more in favour of the reality of a parallax, as these observations are independent of each other, and the maximum and minimum fall in different parts of the year. For it is not impossible—in fact, it has actually happened—that in a single series and even in all the series of observations of the same co-ordinate, made with the same instrument, by the same observer, the errors of observations from some cause or the other followed more or less a law similar to parallax, and led thus to an entirely illusory value of it. All astronomical instruments are exposed to changes of temperature by which their various parts undergo variable and uncertain expansions and contractions, and not only the instruments themselves, but also the piers on which they are mounted—even the very foundations of these piers are liable to these changes, and as their effects have also, more or less, the period of a year, they can often produce very similar effects to parallax, and may be mistaken for them. Refraction itself is subject to mean periodical variations which are more or less dependent on the seasons, and are likewise liable to be confounded with the effect of parallax.

It is therefore highly desirable to use a method for the determination of parallax which should be independent

of such sources of error. Such is the method of differential observations, by which not the right ascension and declination of a star is determined, but merely the difference of its right ascension or declination from that of one of several small stars in its neighbourhood, which are supposed to have no parallax. This method gives really only the difference of the parallax of the two stars observed, but as that of every faint star is undoubtedly exceedingly small, if not altogether imperceptible, this is no practical difficulty. On the other hand, the errors arising from such causes as mentioned before affect the two stars equally, and are therefore entirely eliminated in the result for the relative place. Such observations are made by the most powerful telescopes, with the aid of micrometers; the precision of the observations made with these rendering this method susceptible of extraordinary exactness. When the two stars are simultaneously present in the field of view of the telescope, their relative place is determined by polar co-ordinates, as the micrometer allows the observer not only to measure the apparent distance of the two objects, but also the direction of the line joining these two points with relation to some fixed direction—for instance, that of the parallax, or the declination circle, passing through the middle between the two stars. This angle, reckoned from the northern part of the declination circle, though east, &c., is called the angle of position, and the micrometric apparatus just indicated is called a position micrometer. The co-efficient of the effect of parallax on the distance and the angle of position can be as easily calculated, as for right ascension and declination, and if the apparent distance and angle of position are again expressed by $D + d\,\pi$, $P + p\,\pi$, d and p being these co-efficients, the value of π is deducted from the observations in the same way as was described before. It must be observed, however, that the problem is not quite as simple as it is stated here for the sake of greater lucidity. For parallax is not the only cause which may alter the position of the star in the course of the year, as the star may have also a proper motion, always, however,

rectilinear, whose amount must be determined, at the same time, by the same method by which the parallax is deduced, as well as the corrections of the quantities D and P themselves, which must be supposed as being only approximately known. It is by this method that Bessel first determined the parallax of 61 Cygni, and found for it the value $0''.35$, which, however, has been since somewhat increased by the observations of Struve. The star α Lyræ has also been found to have a parallax of $0''.18$ from the observations made at the observatories at Pulkowa, and at Dublin. The number of stars whose parallax is known with any certainty is still exceedingly limited, owing to the difficulty of these observations; but there is some hope that the many powerful instruments at present in use will contribute also to the advancement of this branch of science.

331. BINARY STARS.—The determination of the parallax of binary stars leads, when their orbits are known, also to the determination of their masses. The binary stars, as mentioned before (Art. 25), are double stars which move in orbits, in the same manner as the planets move round the sun. The orbits of these are elliptical, and the motions of the stars are subject to the same laws as those of the planets. Since, then, the relation of the semi-axes of the orbits and the periodic times determines the relative masses of the central bodies, we can compare the mass of a binary system with that of the sun, whatever the distance.[a] The elements of the orbits of many of these binary stars have been computed, but so far the value of the semi-major axis is known only in seconds, or decimals of seconds.

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[a] If T′ be the time of revolution of a binary system, and T the length of a year, we will have the proportion $T'^2 : T^2 :: \frac{a'^3}{m+m} : \frac{a^3}{M}$; a being the mean distance of the earth from the sun, a' the semi-axis major of the star's orbit; M the mass of the sun, and $m + m$ that of the two stars. Thus the distance of the two stars 61 Cygni subtends at the earth an angle of $15''.5$. While the parallax of this star (or the angle the distance of the earth and sun subtends at the

332. Distribution of Stars.—The stars seem at first sight to be scattered without any law of arrangement; a more careful examination shows, however, that their density, or the number of them which is found in a given space of the heavens, varies regularly. At two points of the celestial sphere diametrically opposed to each other the stars are more thinly scattered than elsewhere, and departing from these points, the density of stars increases in any direction at first slowly, but at greater distances from them more rapidly down to the milky way, when it attains its maximum. Those two points are called the Galactic poles, the great circle to which they belong and which coincides with the average course of the milky way, the Galactic circle. This rapid increase of the stellar density with the Galactic polar distance, in connexion with the bifurcated form of the milky way, led Sir W. Herschel, under the supposition of an approximately uniform distribution of the stars, to the hypothesis that the stars of our firmament, of which our sun forms a part, forms a stratum of definite form, and that its figure resembles roughly a cloven disc or lens. But many features—among others, for instance, the great gap which exists in the widest part of the milky way in the constellation of Argo—cannot be well accounted for by this hypothesis, and will lead yet to a considerable modification of this view.

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star), is 0·″35, consequently the distance between the stars is to the earth's distance from the sun as 1550 is to 35 or as 44·3 : 1. It would appear that its orbit is nearly circular and at right angles to the visual ray, and its period not far from 500 years. Sir John Herschel calculates from these data that the sum of the masses of these stars is 0·353 that of the sun.

CHAPTER XXI.

ON THE DISCOVERIES IN PHYSICAL ASTRONOMY.

333. The astronomical knowledge that existed before the time of Sir Isaac Newton was derived from long and tedious observations, which had been continued through many ages. The various discoveries, such as the elliptical motions of the planets, the law of the periodic times, the precession of the equinoxes, the direct motion of the apogee of the moon's orbit, the retrograde motion of its nodes, the variation and evection of the moon, were apparently so many unconnected circumstances.

It was Newton who first, from a few general laws of matter and motion, by help of mathematical principles, showed the origin and connexion of these different phenomena, and that they were simple results of the general properties which the Creator has ordained should belong to matter and motion. Before his time Physical Astronomy did not exist. The attempts of Kepler, Descartes, and others, to explain several astronomical phenomena from physical principles, now scarcely deserve notice.

334. It would be incompatible with the plan of this work to enter into any detail of the mathematical principles of physical astronomy. But the discoveries in physical, are so connected with plane astronomy, and so important, that it was not possible to avoid the mention of many of them, when occasion offered; and it may not be deemed improper to conclude with a short account of the general advantages which the science of astronomy has received from the application of physical principles.

Sir Isaac Newton has shown that all the bodies of the

solar system mutually attract each other; that the gravitation, or the force of attraction, exerted by or toward any body, is in proportion to the mass of the attracting body; that this force is greater or less according as the distance from the attracting body is less or greater, and that in proportion to the square of the distance.

335. Of the immediate cause of gravitation he confesses himself ignorant. He says[a] that gravity must be caused by an agent acting constantly according to certain laws: but whether this agent be material or immaterial he did not attempt to decide. He reflected much on this subject, but it does not appear that he ever came to any conclusion which satisfied himself. At this day we are not advanced one step farther toward the knowledge of the proximate cause of gravity, than Newton himself had advanced.

The knowledge of the proximate cause, however, is not necessary to ascertain the existence and laws of the action of gravity. The latter are collected from the observation of a variety of facts.

From the laws of the action of gravity, combined with laws of matter and motion, deduced from observations on terrestrial matter, Newton explained the motions observed in the solar system.

The sun, situate in the midst of the planets, attracts them all towards itself, while they also attract the sun; but from the greater mass of the sun, the effect of the planets in moving the sun is very small, compared with the attraction of the sun on the planets.

Had no other impulse been given to each of the planets, they and the sun would have come together in consequence of their mutual attraction. But a proper impulse was given to each planet in a direction either perpendicular, or nearly perpendicular to a line joining the sun and planet. In consequence of this impulse, and of the attraction of the sun, each planet continues

[a] Letter to Dr. Bentley, page 438, vol. 4. Horsley's Edition of Newton's Works.

to revolve round the sun in an elliptical orbit not differing much from a circle—that is, not very eccentric. These impulses must have been given at the creation. They required, to use the words of Newton,[a] "the Divine Arm to impress them according to the tangents to their orbits."

The simple laws of matter and motion, which the Almighty has been pleased to ordain, are sufficient to preserve the motions of the system for a length of time, to which our bounded intelligence cannot put a limit.

336. The preparatory steps of Newton consist, principally, in showing, that a body so projected, while it is at the same time attracted to a fixed centre, describes equal areas in equal times about that centre, and in investigating the laws of the variation of the force by which a body attracted toward a given point may be made to move in a given curve.

He particularly shows, by an interesting application of mathematical principles, that a body moving in an ellipse, and describing equal areas in equal times about one of the foci, must be attracted toward that focus, by a force varying inversely as the square of its distance from the focus: that the squares of the periodic times of bodies, moving in different ellipses about a common centre of force in the common focus, are as the cubes of the greater axes.

He also, conversely, proves that a body attracted to a fixed centre, by a force varying inversely as the square of the distance, and projected in a direction, not passing through the centre, with a velocity not exceeding a certain limit, will describe an ellipse about the fixed centre. The increase or decrease of velocity generated by the attractive force is so exactly combined with the velocity of projection, that the efficacy of the attractive force in drawing it from the tangent of the curve, in which tangent it would continue were the attractive force to cease, is such as always to retain it in the circumference of the ellipse.

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[a] Third Letter to Dr. Bentley.

After considering a variety of cases of motion about a fixed centre, he considers two or more bodies mutually attracting each other.

He also demonstrates that if a sphere consist of particles, each of which attracts with a force varying inversely as the square of the distance, the united forces of all the particles compose a force tending to the centre of the sphere, and varying inversely as the square of the distance from that centre.

337. The application of his investigations to the system of the world may be briefly stated as follows:—

The effort by which all bodies within our reach tend toward the surface of the earth we call *gravity.* If left to themselves, bodies fall toward it in a right line, but, if projected, they tend toward it in a curvilinear course.

By gravity, also, a pendulum, when removed from a vertical position, tends to it again, and so vibrates.

Experiments on the motions of falling bodies and the vibrations of pendulums, after proper allowances made for the resistance of the air, show that this force of gravity, measured by the velocity produced in a given time, is nearly the same in the same place, at any distance from earth's surface to which our experiments can reach.

But along with the knowledge of this fact we also arrive at another of great importance—viz. that however dissimilar bodies are in their visible properties, yet they are all equally affected by gravity, that each particle of a body is acted upon by the same force, that the component parts of air and gold are equally impelled toward the earth. This knowledge is derived from observing that all bodies, at the same place, describe, in falling from rest towards the earth, equal spaces in the same time, the resistance of the air being removed.

To these laws of gravity we are enabled, also by experiment, to add a third—that the gravitation toward the earth is the united effect of gravitation toward its separate parts, or that each particle of matter attracts; from whence it follows that the attraction of gravita-

tion between terrestrial matter is mutual. Several strong arguments induced Newton to adopt this as an hypothesis, but it seems not to have been fully verified till long after his death. No facts, proving it, were known to him.

If the earth had been originally a fluid of uniform density, it would have followed from the mutual attraction of its parts, and from its rotation on its axis, that the increase of the length of a pendulum vibrating seconds would have been nearly as the square of the sine of latitude. Also, if the earth had been originally a fluid of unequal density, the denser parts would have so arranged themselves towards the centre that the law of increase of the length of the pendulum would still be as the square of the sine of latitude. Now we know that the interior of the earth is denser than the surface, and a great number of experiments have shown that in both hemispheres the increase of the length of the pendulum is as the square of the sine of latitude. The ellipticity of the earth also agrees with that of the oblate spheroid, which a fluid sphere would assume if it revolved round an axis in the same time as the earth does. From hence it has been inferred that the earth was originally in a fluid state.

Laplace has shown that the effect of the attraction of the excess of matter at the equator causes two equations in the moon's motion, one in latitude and the other in longitude. The quantities of these equations, having both been well ascertained by an examination of a very great number of observations, have served to deduce the excess of the equatorial above the polar diameter. Each equation gives the same excess very nearly—viz., $\frac{1}{305}$. Laplace also has shown that the excess of matter at the equator of Jupiter occasions certain equations in the motions of the satellites.

338. Before the time of Newton, it appears that several eminent men had notions respecting a mutual attraction in the system. Kepler, in his work "De Stellâ Marte," speaks of the mutual gravitation of the earth and moon. He says that if they were not re-

tained at their proper distances, the earth and moon would come together, the moon coming over 53 parts of the distance, and the earth over one part. He also seems aware, that not only the tides are caused by the attraction of the moon, but also that the irregularities of the moon are caused by the united actions of the sun and earth. But it does not appear that either he, or any other person before Newton, had an idea that the force of gravity toward the earth, combined with the projectile velocity, retained the moon in her orbit, or any notion of the variation of gravity at different distances from the earth.[a]

Kepler, although an excellent mathematician, seems not to have been able to apply that science to his ideas of gravitation, and Galileo had the merit of first applying the principles of mathematics to investigate the effects of gravity at the earth's surface. He first showed that a projectile acted upon by the uniform force of gravity, in parallel lines, describes a parabola.

We find no mathematician between him and Newton pushing the inquiry farther, and investigating the curve in case of a projectile taking such a range that gravity could no longer be considered to act in parallel lines. About the time, however, that Newton applied himself to these inquiries, we see several mathematicians considering the laws of action by which bodies may revolve with uniform velocities in different circles about the same centre.

We are told that an accidental circumstance first led Newton to consider the effects of the gravity of the earth at a distance from the surface, and to inquire whether that gravity did not extend to and retain the moon in her orbit: the moon, by the continual action of this force, being drawn from a rectilinear course, and made to revolve about the earth in a nearly circular orbit.

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[a] It may be considered as a curious circumstance that Galileo computes how long a body would take to fall from the moon to the earth. He supposes the force of gravity to continue the same throughout the whole distance.

By arguments similar to those which have been already given in Art. 308, he concluded that the force of gravity, diminished in the duplicate proportion of the semidiameter of the earth to the moon's distance, was the force acting at the moon and retaining it in its orbit.

339. We deduce somewhat more easily the law of gravitation towards each of the planets which have satellites. It is found that the satellites of Jupiter move round Jupiter in orbits nearly circular, and that the squares of the periodic times are as the cubes of their distances from the primary. Whence it may be easily shown that they are constantly impelled toward or attracted by Jupiter, by a force increasing as the square of the distance from Jupiter decreases. The same may be said of Saturn and Uranus. Here, then, are the earth, Jupiter, Saturn, and Uranus, each attended with an attractive influence, acting by the same laws, and therefore, by analogy, we may justly conclude that the remaining planets attract by the same laws.

340. Newton's investigations of the motions of bodies about the same centre of force, combined with Kepler's discoveries, prove that each of the planets is attracted toward the sun, by a force varying inversely as the square of the distance from the sun.

For Kepler showed from observations, 1st, that each planet described equal areas in equal times about the sun; 2nd, that it moved in an ellipse of which the sun occupied the focus; and, 3rd, that the squares of their periodic times were as the cubes of the greater axes of their orbits. Newton demonstrates by geometry that when the first takes place the planets must be retained in their orbits by a force tending to the sun; when the second is true, that that force must vary as the inverse square of the distance; and that when the third holds good, the unit of attractive force must be the same for all the planets.

341. Thus then by the moon we ascertain that the earth exerts an attractive influence; by the satellites of Jupiter, Saturn, and Uranus, that these planets

exert a similar influence: and by the forms of the planetary orbits and laws of motion in those orbits, that the sun also possesses an attractive force. We find the law of action is the same in all the attracting bodies. But if we examine farther we find the forces exerted very different at the same distance from each body. If we compute the force exerted by the earth, at the distance of the sun, by diminishing the force of gravity at the earth's surface in the duplicate proportion of the semidiameter of the earth to the sun's distance, we shall find it small indeed, compared with the force the sun exerts on the earth.

342. As the planets are attracted toward the sun and attract their satellites, we may conclude that they attract one another and the sun. Also, as we find the attraction of the earth made up of the attractions of its parts, so we may conclude the attractions of the sun and planets composed of the attractions of their parts, and that the law in the system is, that every body attracts with a force, at a given distance, in proportion to its mass, and that the force diminishes as the square of the distance of the attracted body is increased. This has been wonderfully confirmed in recent years by the discovery of Neptune. The path of Uranus had been calculated, allowing for the disturbing action of Jupiter and Saturn; but this path was found not to agree strictly with former observations of Uranus when it was supposed to be a fixed star, nor with recent observations since it was recognised as a planet, in 1781. From 1690 to 1715 its observed place was in advance of its computed; from 1715 to 1771 it was behind; from 1780 to 1828 it was in advance by a quantity which increased from 3″·45 to a maximum of 24″·16, in 1804; after which it again diminished till its observed and computed places agreed in 1830. Since which time it has been behind its computed place. Adams and Leverrier independently undertook the mathematical solution of this problem; assuming the universality of gravitation, to determine the mass and position of an unknown planet which could, by its at-

traction on Uranus, produce an effect not accounted for by the perturbations of Jupiter and Saturn; that is, being given the effect produced, to discover mathematically the cause. Both succeeded in the attempt, and Leverrier was the first to publish his results, which indicated the exact place where Neptune was to be found.

343. As soon as Newton had published his discoveries, there could be no rational doubt of this being the law which exists throughout the solar system. Recent investigations have shown, as has been already stated, that the law of mutual attraction, according to the inverse square of the distance, extends even to binary stars; every step, in fact, that has since been made in Physical Astronomy has furnished additional proofs of the universality of the law of gravitation.

It is probable that Newton derived no assistance in the discovery of the law of gravity: yet he does not seem unwilling that others should have a share in the merit. He ingenuously tells us that Wren, Hook, and Halley had separately discovered, from Kepler's law of the periodic times, the law of attraction towards the sun, if the planets moved in circular orbits. But the great fame of Newton rests not upon this foundation, that he merely discovered the law of gravity. He proceeded by synthesis to examine the phenomena that would offer themselves in a system so regulated. His transcendent mathematical powers enabled him to point out the origin of all the more splendid discoveries of former ages. He showed that the planetary orbits must be elliptical, that the lunar irregularities, the precession of the equinoxes, and the phenomena of the tides must take place from the principle and law of universal attraction; thereby evincing, in the strongest manner, that he had arrived at the knowledge of those laws which the Creator had willed for upholding the system of the world.

344. Newton had extended the boundaries of mathematical knowledge, as much as he had those of physical. He preferred exhibiting his investigations and conclusions in a geometric, rather than in an analytic form, as

better suited to the general outlines of physical astronomy, and also as better adapted to call the attention of the world to his great discoveries. To extend the limits of physical astronomy, and to explain discoveries that have been made by comparing modern and ancient observations, it has been found necessary to adopt entirely the analytic method.

A considerable time elapsed from the publication of Newton's Principia, in 1687, before any attempt was made to extend the investigations of Newton. In 1740 Maclaurin, Euler, and Bernouilli shared a prize given by the Royal Academy of Sciences at Paris, for their dissertations on the tides, in which they made considerable advances in the path pointed out by Newton.

Soon after, Euler, D'Alembert, and Clairaut engaged in the famous problem of the three bodies, as it has been called—that is, to investigate the motions of three bodies, acting upon each other according to the laws of gravity. The problem, in its general extent, is beyond the powers of analysis in its present state: but we can approximate to the solution with sufficient exactness when the disturbing force of the third body on the other two is small in comparison to their mutual attraction. Thus, the sun disturbs the motions of the moon, as seen from the earth, only by the difference of its attractions on the moon and earth, which difference is always very small, compared with the force by which the moon is attracted towards the earth.

The importance of an exact knowledge of the lunar motions, in finding the longitude at sea, seems principally to have incited the exertions of these mathematicians. A difficulty soon occurred which made them at first doubt of the exactness of the Newtonian law of gravity. They could not reconcile the mean motion of the lunar apogee, as determined by calculation, with that deduced from observation.[a] They saw the latter

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[a] Newton himself seems to have long been sensible of this difficulty, and to have exerted himself in the computation without success. In the first edition of the Principia he mentions computations by which

was double of the former. At last Clairaut, by extending his approximations[b] overcame this difficulty, and added a new proof of the law of gravity.

345. There were two phenomena, however, to which Flamstead and Halley first called the attention of astronomers, which for many years baffled all attempts to account for them, from the received laws of gravity. These were the acceleration of the moon's motion (Art. 264), and the acceleration of Jupiter's, and retardation of Saturn's motions (Art. 245).

Dr. Halley's computations on the ancient observations had been verified by other astronomers, and no doubt remained of the facts. The acceleration of the moon's motion had also been verified by computations made on three eclipses observed by Ibn Junis, near Cairo, towards the end of the 10th century.

346. Euler, who, as a mathematician, ranks so high, directed his attention to the motions of Jupiter and Saturn. So early as the year 1748 he published an investigation of them, but failed to explain the difficulty. Other mathematicians engaged in the inquiry. For a long time the object of their pursuit eluded them, but their exertions tended much towards perfecting physical astronomy.

Euler investigated many of the disturbances which

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he had ascertained the agreement nearly of his Theory with Flamstead's Tables, accommodated to the Hypothesis of Horrox. But he says, "Computationes autem ut nimis perplexas et approximationibus impeditas, neque accuratas, apponere non lubet." In the subsequent editions of the Principia, he does not attempt to reconcile the observed motion of the apogee with his Theory. It would be very interesting to know the particulars of his computation.

[b] Not considering the eccentricity and inclination of the lunar orbit, the mean motion of the lunar apogee, that of the moon being unity, is expressed by a series of terms of the form $\frac{3}{4}m^2 + \frac{325}{32}m^3 +$ &c., were $m = \frac{\text{periodic time of the moon}}{\text{periodic time of the sun.}} = \frac{1}{13}$ nearly. Clairaut's first approximation extended only to the term $\frac{3}{4}m^2$.

take place by the mutual action of the sun and planets, He first showed that the diminution of the inclination of the ecliptic to the equator, which ancient observations appeared to show, was occasioned by the action of the planets by which the plane of the earth's orbit is gradually changed.

Lagrange, who has become so distinguished by his many splendid improvements in mathematics and mathematical philosophy, about 1765 published[a] his investigations respecting the motions of Jupiter and Saturn.

The celebrated Laplace, in 1773, showed that the mean motions and mean distances of the planets were not subject to any variation arising from their mutual actions on each other, or at least were so nearly constant that nothing could appear to the contrary from the most ancient observations. Hence the explanation of the acceleration of Jupiter and retardation of Saturn, that Lagrange and others had given, could not be the true one.

Soon after Lagrange himself proved strictly, what Laplace had proved only by approximation, that neither the mean motions nor mean distances of the planets were subject to any perceptible alteration from their mutual attraction.

It was not till 1786 that Laplace discovered the true explanation of the difficulties as to Jupiter and Saturn, after it had been sought for in vain above thirty years, by the continued exertions of the first mathematicians. His investigations furnished another confirmation of the mutual attraction of the system. He showed that the quantity of acceleration in the motion of Jupiter and retardation in that of Saturn, deduced from computation, agreed with observation.[b]

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[a] Turin Memoirs, vol. 3.

[b] It is not easy to make his discovery intelligible to those not conversant in the computations of physical astronomy. The *equations* arising from the mutual attraction of the bodies of the system are

347. The difficulty of the acceleration of the moon's motion yet remained, and it had fully as much occupied the attention of those who endeavoured to improve Physical Astronomy as that just mentioned.

The Royal Academy of Sciences at Paris had proposed it several times as the subject of their prize. It had eluded the researches of Lagrange, yet his investigations on the subject gained the prize in 1772. Bossut had endeavoured to explain it by the resistance of an ether, or subtle fluid pervading the whole system. Laplace endeavoured to explain it by supposing that the transmission of gravity, like that of light, was not instan-

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divided into *secular* and *periodical*. In fact it is now known that all equations are periodical, but the term "secular" distinguishes those that do not depend on the position of the *bodies* as to each other. As these equations appertain to a long period, they are called secular. Periodical equations are those that depend on the position of the bodies to each other.

Thus the *Variation* of the moon (Art. 265) depends on the angular distance of the moon from the sun, being proportional to the sine of twice the angular distance of the moon from the sun, and is called a *periodical* equation. The acceleration of the moon's motion, not depending on the position of the sun, moon, and earth, is called a *secular* equation.

Lagrange had at first conceived that the acceleration of Jupiter was from a secular equation; but Laplace, and then he himself, showed that no such equation could exist in the planetary motions. Therefore Laplace was led to look for a periodical equation, and he observed that as twice the mean motion of Jupiter was very nearly equal to five times that of Saturn, an equation of a very long period would result from thence, which might be sensible. To investigate this, it was necessary to extend the approximations to a greater length than had hitherto been done. It might, and it is likely it did, occur to others before this time, that this was a probable source of the phenomena, but till the existence of a secular equation had been disproved, the formidable calculations might have deterred them. Similarly, thirteen times the period of Venus very nearly equal eight times that of the earth, which leads to an irregularity in the earth's motion, detected by the calculations of Mr. Airy, which has a period of 240 years, and amounts only to a few seconds at its maximum. This is dependent upon powers and products of the eccentricity and inclination of the fifth order. Equations depending on the difference between five times the motion of any planet and twice that of another actually exist, but are insensible in the other planets.

taneous, and on this hypothesis he made some important investigations. At length, in 1787, Laplace himself discovered, as his method showed, that it was a simple result of the laws of gravity. The actions of the planets, besides changing the plane of the earth's orbit, change also its eccentricity. The eccentricity now is diminishing, and will continue, for many ages to come, to do so. It will afterwards increase, and thus be subject to periodical changes. These changes will affect, through the action of the sun, the angular velocity of the moon about the earth, and hence at the present time an acceleration takes place. Nothing seemed to be more satisfactory than the results of Laplace on this subject. He found that his value of the acceleration of the moon's motion was in accordance with the observation of the ancient eclipses, and pointed out in consequence that this accordance between theory and observation in respect to this secular acceleration proves the constancy of the length of the day; for supposing this length to be greater now than it was in the days of Hipparchus by the hundredth part of a second the actual duration of a century would be greater by $365\frac{1}{4}$ seconds, during which the moon describes an arc of $173''\cdot2$. The mean secular acceleration would in consequence be increased by a larger quantity than would be admissible. We have seen already (Art. 264) that Professor Adams pointed out a deficiency in Laplace's method, and has shown that the true value of the acceleration produced by this cause is only one-half of that which Laplace had calculated. The deficiency must be accounted for by other causes. At the same time Laplace's argument for the constancy of the length of the day falls to the ground. Indeed the natural explanation which now presents itself to account for the discrepancy of the theoretical value and that derived from observations is that the portion of the moon's acceleration which is not accounted for theoretically is only apparent, and arises from a change in the length of the sidereal day, or the period of the earth's rotation, which we must assume to be increased $\frac{12}{1000}$ of a second in the course of 2000

years, in order to bring the theoretical value of the acceleration of the moon's motion in agreement with the observation of ancient eclipses. This, as has been already stated, may possibly be owing to the friction of the tidal wave, which may tend to retard the earth's rotation.

348. The results of all the improvements in physical astronomy, since Newton first called the attention of mankind to it, have been given to the world by Laplace, in his great work, entitled "Mecanique Celeste." This and the Principia of Newton will probably be considered by late posterity as the two noblest monuments of human science. The Principia of Newton was the work of one mind, which could derive no assistance from those who had gone before. The energies of the most distinguished abilities had been for many years employed in collecting materials for the fabric that Laplace has erected. Newton and Lagrange have assisted in an eminent degree; Maclaurin, Euler, T. Simpson, Clairaut, D'Alembert, and others, greatly contributed. Laplace himself, besides the merit of planning, and of selecting, and arranging the materials, has the honour of having executed many of the most difficult and highly finished parts of this great work.

349. No motion is now known to exist in the system but what we can show to be conformable to the laws of universal gravitation. It must not, however, be supposed that the analytical science, as applied to physical astronomy, is perfect, or even in a state approaching to perfection. Notwithstanding the great progress that has been made during the last century, much remains to be done. Because the orbits of the planets are inclined at small angles to the ecliptic and to each other, and because the eccentricities of the orbits are small, we are enabled with tolerable facility to compute by approximation the disturbing effects of the planets on each other. But it is a work of great labour and difficulty to compute the disturbances of bodies like the asteroids which move in orbits of greater eccentricity and inclination, although these calculations have been

much simplified, specially by the labours of Professor Hansen of Gotha, in this case also. But in many cases where the orbits are very eccentric, for example, in respect to comets, the only way of computing their perturbations is to follow the comet and disturbing body, step by step, and to compute at sufficiently short intervals the perturbations which the comet undergoes owing to their relative position.

The mean motions and mean distances of all the planets are to be considered invariable, and the effects of their mutual actions are all periodical. We can now ascertain for thousands of years the state of the system, should such a continuance be permitted by the Divine Author.

The obliquity of the ecliptic, which now is diminishing by a small quantity every year, will never be diminished more than a degree or two. This is a very interesting result. Had the obliquity continued to decrease, the equator at last would have coincided with the ecliptic, the change of seasons would have no longer existed, and a great part of the earth would have been thus rendered incapable of producing the necessary food for the existence of men and other animals.

350. In all our inquiries into the operations of nature, by which should always be understood the modes of existence and laws assigned to the objects of creation by the Divine Creator, we meet with sources of delight and admiration; but in none more than when we contemplate the objects of astronomy.

The magnitudes and the distances of the bodies of the solar system, the immense number of the fixed stars placed at immeasurable distances from us, and from each other, show us the magnificence of the creation.

By the discoveries of Newton we are permitted, as it were, to understand some of the counsels of the Almighty. From these we can, by demonstratiou, overturn the absurd doctrine of blind chance. We see that a Supreme intelligence placed and put in motion the planets about the sun in the centre, and ordained the laws of gravitation, having provided against the smallest

imperfection that might arise from time. And let us not imagine that only in these vast bodies the Supreme care was employed. Let us not imagine that man, apparently so insignificant, cannot be an object of attention in a world so vast. The protecting hand of the Creator is equally visible in the smallest insects and vegetables, as in the stupendous fabrics which astronomy points out to us. He who formed the human mind so different in its powers and mode of existence from the rest of the works of creation, has assigned laws peculiarly suited to *its* preservation and improvement: laws not mechanical, but moral: laws only obscurely seen by the light of reason, but fully illumined by that of Revelation.

QUESTIONS FOR EXAMINATION.

N. B.—The number after each question indicates the article in which the answer may be found.

CHAP. I.

1. Enumerate the planets.
2. Define the horizon, meridian, altitude, azimuth, zenith, nadir, equator, ecliptic, right ascension, declination, vertical circles, prime vertical, obliquity of the ecliptic, tropics, solstices, zodiac, longitude, latitude. What are the right ascensions and declinations of the sun on the following days:—March 20; June 21; September 23; December 22?
3. Where is the constellation Aries now in relation to the intersection of the ecliptic and equator, and why is it removed? 13.
4. What is meant by a sidereal day, and a sidereal hour? 14. How would you find the length of a sidereal day without the use of a telescope? 7. Why is the interval between two successive transits of the sun or moon greater than 23 hours 56 minutes? 9, 10. Why is the interval between two oppositions of the moon greater than that between two successive returns to the same fixed star? 10.
5. Why is the altitude of a celestial body greatest when on the meridian? 8.
6. Being given the right ascension and declination of a star, how may its latitude and longitude be found? 15.

CHAP. III.

7. Give the arguments which prove that the earth is a sphere. 31.
8. If the earth be a sphere, the change in the altitude of the pole will be proportional to the space gone over north or south. Prove this? 34.
9. Length of a degree on the earth's surface? 35.
10. How is the diameter of the earth found from this? 35.

11. Difference between equatorial and polar diameters? 35, *note.*
12. What is meant by the ellipticity of the earth? its value? 35, *note.*
13. Eratosthenes' method of determining the size of the earth? 36, *note.*
14. What is meant by the terrestrial and the celestial equator? 36.
15. Difference between terrestrial and celestial latitude and longitude? 36; 15.
16. Prove that the altitude of the pole is equal to the latitude of the place. 39.
17. Why will the heavenly horizon be a great circle? 40.
18. Why are the days longer than the nights, in Summer, in northern latitudes, and the opposite in Winter? Show by a diagram how the change in the declination of the sun produces the succession of the seasons. 11, Chap. I. Why is Summer longer than Winter? and why is the heat not greatest on the longest day? 11, Chap. I. 41.
19. What is meant by an oblique sphere? a right sphere? a parallel sphere? 41, 42, 43.
20. What is the lowest latitude at which the day can be 24 hours long? and why?
21. Why are day and night always equal at the earth's equator? 42. Show that the sun will be vertical to a place within the tropics on two days in the year? What will be its declination on those days?

CHAP. IV.

22. Explain atmospheric refraction? 46.
23. Why does it increase a star's altitude? 46. Where is it greatest? 47.
24. What is the *law* of atmospheric refraction? 48.
25. Prove that the refraction equals $(\mu - 1) \tan Z$. 48.
26. How is the *coefficient* of refraction found when the latitude of the place is known? 49. If the greatest and least true zenith distances of the pole star be $52° 7' 30''$, and $48° 32' 28''$, find the latitude of the place. 45.
27. Bradley's method of finding the coefficient of refraction when the height of the pole is not known? 50.
28. Cause of twilight? when does it begin and end? 52. How may its duration be found? Show this by a figure. 53.
29. Why is twilight shortest at the equator? 53.
30. When does twilight last all night at a given latitude? 53. Find sun's declination when the twilight begins to last all night at lat. $53° 18' 53''$.
31. Explain the oval appearances of the sun and moon when setting? 54.

CHAP. V.

32. How are the apparent diameters of the sun, moon, and planets measured? 57.

33. In what manner may the difference of declination of two stars which have nearly the same declination be measured? 57.

34. If the stars in this case differ much in right ascension the circumstances are altered: why? 57.

35. How is the angle found which two places on the same terrestrial meridian subtend at a planet? 58.

36. How are the circumstances of this observation affected when the two places are not under the same meridian, and when the star and planet are not on the meridian together? 58, *note.*

37. How is the angle which the earth's disc subtends at a planet found? 59.

38. Prove the formula for the horizontal parallax of the moon in terms of the meridian zenith distances of the moon, as seen from two obervatories in given latitudes and in the same longitude? 59.

39. Why does this method fail in the case of Jupiter? how is the distance of that planet found? 60. From two observations of Jupiter in successive quadratures of the planet, the angle which the earth's orbit subtends at that planet may be found.

40. Being given the horizontal parallax of a heavenly body and the diameter of the earth, how is its distance from the earth found? 64, *note.*

41. By what method are the diameters of the sun, moon, and planets in miles found? 63.

42. How is the rotation of the sun on its axis proved? 65.

43. What appearances prove that the sun's spots are not dark bodies revolving round him? 65.

44. What is meant by diurnal parallax? When is it greatest? and why? How does it vary with the altitude? 74.

45. What effect has parallax on the position of a planet? and on that of a fixed star? and why? 75.

CHAP. VI.

46. What are the arguments for the diurnal motion of the earth? 76.

47. What is the force of the argument from analogy? 77.

48. State the argument from centrifugal force? 79.

49. What is the experiment of a falling body which is adduced in favour of the earth's rotation? 80.

50. Explain the effects which the rotation of the earth would produce in the plane of vibration of a pendulum—(1) at the pole; (2) at the equator; (3) at an intermediate latitude. 81.

51. What are the arguments in favour of the annual motion of the earth? 83.

52. State the argument from analogy. 84.

53. What is the argument from universal gravitation? 85.

54. What effect would be produced upon the seasons were the earth's axis to lie in the plane of the ecliptic, or to be perpendicular to it? 86. If the rotation of the earth on its axis were from east to west, but in the same time as at present, what change would be produced in the length of the mean solar day, and in the number of days in the year? 9, 76, &c.

55. Describe the position and motion of the earth's axis. 86.

56. How does this explain the change of seasons? 87.

57. What is meant by the annual parallax of a star? Why is it so small? 88.

58. How may we find a minor limit to the distance of γ Draconis? 88, *note*.

59. Show that, if we select a star in the solstitial colure, the annual parallax will be known, if the difference between the greatest and least polar distance be observed. 88, *note*.

60. What was Bessel's method of finding the parallax of a star? 88.

61. What is the star nearest to the earth, and its parallax? 88.

62. Define the precession of the equinoxes. What is its amount? 89. Why so called? What effect has it on the longitude of a star?

63. What is the physical cause of precession? 90.

64. What is nutation? 90.

65. How may precession and nutation be graphically represented? 91.

CHAP. VII.

66. How may it be shown that the motion of the planets round the sun, in orbits nearly circular, accounts for the observed phenomena? 94, &c.

67. What is meant by the elongation of a planet from the sun? What are the *inferior* planets? When is their elongation greatest? How does the greatest elongation determine the ratio of the distance of the earth and planet from the sun? 95.

68. When is a planet said to be in *inferior* or *superior* conjunction? 95.

69. By noting the time from last inferior conjunction, and the elongation of an inferior planet from the sun, the ratio of the distances of the earth and planet from the sun may be determined. 96.

70. How may we deduce the angular distance of a planet from the sun? 96, *note*.

71. Knowing the ratio of the distances of the earth and planet from the sun, how may we calculate what the elongation ought to be at any definite time? 96.

72. How are the retrograde and stationary appearances of an inferior planet explained? 97. And how are those of a superior planet? 101.

73. Why is a planet retrograde at inferior conjunction or opposition? 98. In what positions will its motion be direct?

74. How is the periodic time of an inferior planet found? 103.

75. And in what way is that of a superior? 104.

76. What are the *ascending* and *descending nodes* of a planet's orbit? 105.

77. How is it shown that Mercury and Venus derive their light from the sun? 105.

78. What are the changes in the appearances of these planets? 105. And how are they explained? 106.

79. To what is the part of the illuminated surface of a planet which is seen by a spectator on the earth proportional? and why? 106.

80. Prove that the illuminated surface of a superior planet which is visible to us is least at quadratures. 108.

81. Why does Mars appear gibbous to us, and never crescent-shaped? 108.

82. Jupiter never appears either gibbous or crescent-shaped: why? 108.

83. Upon what does the brightness of a planet depend? 109.

84. Why are the inferior planets not brightest at superior or inferior conjunction? In what part of their orbits are they brightest? 109.

85. When is Venus a morning star? And when an evening star? And why? 110.

86. Why is Mercury seldom seen by the naked eye? What are the best circumstances for observing that planet? 110.

87. What relation did Kepler discover to exist between the periodic times and the distances of the planets from the sun? 113. The distance of Mars from the sun being to that of the earth as 1·524 : 1, find his periodic time. Find it from the consideration that the interval between two successive oppositions is about 780 days.

88. What is Bode's law of planetary distances? 114.

89. This empiric law fails in the case of one of the planets? 114, *note*.

90. What was the Ptolemaic system? 115.

91. Prove that the velocities of the planets are inversely as the square roots of their distances from the sun. *Page* 88, *note*.

92. Hence the stationary points may be graphically represented. *Page* 89.

CHAP. VIII.

93. How may it be shown that Jupiter is an opaque body? and also that his satellites are opaque? 116.

94. Whence does it appear that their orbits are inclined to that of Jupiter? 116.

95. Why do we not see both the beginnings and the endings of the eclipses of the first two satellites? 117.

96. What are the meanings of an *eclipse*, an *occultation*, and a *transit* of a satellite? 117.

97. What remarkable relation exists between the mean longitudes of the first three satellites? How does it follow that they cannot be all eclipsed at once? 119, *note.*

98. Who discovered these satellites? Why was this discovery at first disappointing? 122.

99. What scientific discoveries followed in the wake of this? 122.

100. What is the mean inclination of the Moon's orbit to the ecliptic? What is her periodic time? How is it shown that she does not describe an exact circle round the earth? What is her average apparent motion in one hour? 125.

101. What is the motion of the moon's nodes? 126.

102. What proves that the moon is illuminated by the sun? 127.

103. When is the moon new, full, crescent-shaped, gibbous? 127.

104. Prove that the enlightened part varies as the versed sine of the angle of elongation, nearly. 128.

105. How is the moon's periodic time found? 129.

106. What is the Metonic cycle? What are the *golden numbers*, and why so called? 130.

107. Describe the appearance of the moon a few days before and after new moon, and explain its cause? 131.

108. What method did Aristarchus of Samos use for ascertaining the sun's distance from observations of the moon? 132.

109. Why may we conclude that there are no seas on the moon? 133.

110. What is the method adopted for the purpose of measuring the height of a mountain on the moon? 135.

111. How may it be inferred that the lunar mountains are *relatively* three times higher than those of the earth? 135, *note.*

112. Describe the moon's *librations* and their *causes?* 136.

113. Why is it that there is little moonlight in Summer, and a good deal in Winter? 137.

114. When is the moon's retardation in rising least? and when greatest? and why? 138.

115. What is the phenomenon of the Harvest Moon? 139.

116. Explain this phenomenon? 140.

117. What is known of the atmospheres of the planets? 141.
118. How may it be shown that the moon's atmosphere, if any, must be extremely rare? 142.
119. Under what circumstances are Saturn's rings invisible to us? 143.
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N.B.—The references are to the *articles*, not to the pages.

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